Math 171 - Extra Credit

December 3, 2013

Name key

Score

Show all work to receive full credit. This problem is worth 5 extra credit points and is due on Thursday, December 5.

Use the definition of the definite integral as a limit of Riemann sums to evaluate

$$\int_0^2 (x^2 - x + 1)\,dx.$$

In order to simplify the finite sums that will arise, you may use either the formulas on page 255 (of the textbook) or a computer algebra system (such as WolframAlpha).

$$\Delta x = \frac{2-0}{N} = \frac{2}{N}, \quad f(x) = x^2 - x + 1$$

PARTITION: $0 < \frac{2}{N} < \frac{4}{N} < \cdots < \frac{2k}{N} < \cdots < \frac{2N}{N}$

USING RIGHT SUBINTERVAL ENDPOINTS,

A RIEMANN SUM IS $\displaystyle\sum_{k=1}^{N} f\left(\frac{2k}{N}\right)\Delta x = \sum_{k=1}^{N}\left(\frac{4k^2}{N^2} - \frac{2k}{N} + 1\right)\left(\frac{2}{N}\right)$

USING WOLFRAM ALPHA, THIS SIMPLIFIES TO

$$\frac{8N^2 + 6N + 4}{3N^2}.$$

$$\int_0^2 (x^2 - x + 1)\,dx = \lim_{|\Delta x| \to 0} \sum_{k=1}^{N} f\left(\frac{2k}{N}\right)\Delta x$$

$$= \lim_{N\to\infty} \frac{8N^2 + 6N + 4}{3N^2} = \boxed{\frac{8}{3}}$$