Math 171 - Test 1

September 19, 2013

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (6 points) Use a table of values to estimate the following limit. Your table must show function values at six or more points.

$$\lim_{x \to 0} \frac{5^x - 1}{\sin x}$$

X	$(5^x - 1)/\sin x$
0.1	1.7491
0.01	1.6225
0.001	1.6107
0.0001	1.6096
−0.01	1.5966
−0.001	1.6081
−0.0001	1.6093

IT LOOKS LIKE

$$\lim_{x \to 0} \frac{5^x - 1}{\sin x} \approx 1.61$$

2. (5 points) Determine whether each statement is true (T) or false (F).

(a) F $\lim_{x \to 0} \sqrt{x} = 0$ $\qquad \lim_{x \to 0^+} \sqrt{x} = 0$

(b) F If $g(2) = 1$, then $\lim_{x \to 2} g(x) = 1$.

(c) T If $\lim_{x \to 7} f(x) = f(7)$, then f is continuous at $x = 7$.

(d) F If $\lim_{t \to 0} f(t) = -10$, then $f(0) = -10$.

(e) F If f is not defined at $x = c$, then $\lim_{x \to c} f(x)$ does not exist.

3. (4 points) Explain why $\lim_{x \to 1} \dfrac{x^2 - x}{|x - 1|}$ fails to exist.

$$\frac{x^2 - x}{|x-1|} = \frac{(x-1)(x)}{|x-1|} = \begin{cases} -x, & x < 1 \\ x, & x > 1 \end{cases}$$

$$\underbrace{\lim_{x \to 1^-} \frac{x(x-1)}{|x-1|} = -1 \quad , \quad \lim_{x \to 1^+} \frac{x(x-1)}{|x-1|} = +1}_{\text{LIMIT AT } x = 1 \text{ DNE}}$$

4. (10 points) The table below gives values of the **continuous** functions f and g at selected points.

x	-2	-1	0	1	2
$f(x)$	1.5	2.75	2	-1.75	-3.4
$g(x)$	-4	-6.1	-2	1.5	2.25

(a) Find $\lim_{x\to 2^-} f(x)$. Explain your reasoning.

SINCE f IS CONTINUOUS AT $X=2$

$\lim_{x\to 2} f(x) = f(2)$. $\lim_{x\to 2^-} f(x) = f(2) = \boxed{-3.4}$

(b) Find $\lim_{x\to 0} f(g(x))$.

CONTINUITY $\Rightarrow$ LIMIT $= f(g(0)) = f(-2) = \boxed{1.5}$

(c) Find $g(3)$ if $\lim_{x\to 3} g(x) = 4.9$.

$g(3) = \boxed{4.9}$

(d) Find an interval on which you can be sure the equation $f(x) = 2.25$ has at least one solution. Explain your reasoning.

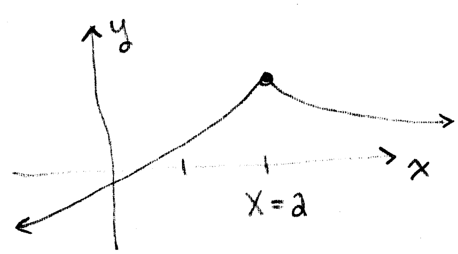
$f(x) = 2.25$ SOMEWHERE BETWEEN $X=-1$, WHERE $f(-1) > 2.25$, AND $X=0$, WHERE $f(0) < 2.25$. SINCE f IS CONT, IT TAKES ON EVERY VALUE BETWEEN $f(-1)$ AND $f(0)$.

(e) (Bonus: 1 point) What is the name of the theorem you used in part (d)?

INTERMEDIATE VALUE THM

5. (4 points) Suppose you have the graph of the continuous function f. Explain what the graph might look like at $x = 2$ if $f'(2)$ does not exist.

THE GRAPH WOULD HAVE A SHARP POINT.



6. (6 points) Find and classify the discontinuites of $R(x) = \frac{x^2+2x-8}{x^2-x-2} = \frac{(X+4)(X-2)}{(X+1)(X-2)}$

R IS DISCONT WHERE IT IS NOT DEFINED: $X=-1$, $X=2$

INFINITE REMOVEABLE

DISCONT AT $X=-1$ IS AN INFINITE DISCONT (NON REMOVEABLE) SINCE $\lim_{X\to -1} R(x)$ DNE

DISCONT AT $X=2$ IS REMOVEABLE SINCE $\lim_{X\to 2} R(x) = \lim_{X\to 2} \frac{X+4}{X+1} = \frac{6}{3} = 2$

7. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{t\to 0} \frac{\sin 3t}{5t} = \frac{1}{5} \cdot \frac{3}{1} \lim_{t\to 0} \frac{\sin 3t}{3t} = \boxed{\frac{3}{5}}$

0/0 MORE WORK

(b) $\lim_{r\to 0^+} \frac{\sqrt{2+r}-\sqrt{2}}{r} \cdot \frac{\sqrt{2+r}+\sqrt{2}}{\sqrt{2+r}+\sqrt{2}} = \lim_{r\to 0^+} \frac{\cancel{2}+r-\cancel{2}}{r(\sqrt{2+r}+\sqrt{2})}$

$= \lim_{r\to 0^+} \frac{1}{\sqrt{2+r}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$

(c) $\lim_{x\to 4^-} \left(\frac{x-4}{x^2-8x+16}\right)$

1/0 UNBOUNDED

$= \lim_{X\to 4^-} \frac{1}{X-4}$ $\quad \frac{+}{-} = -$

$\boxed{\text{LIMIT IS } -\infty}$

(d) $\lim_{x\to 6} \frac{(x-4)^2+2(x+1)}{x+3}$

$= \frac{2^2+2(7)}{9} = \frac{18}{9} = \boxed{2}$

8. (6 points) Is g continuous everywhere? Carefully explain your reasoning.

$$g(x) = \begin{cases} x^2+2, & x \le 0 \\ 1+\cos x, & 0 < x < \pi \\ 1 + x\sin x, & x \ge \pi \end{cases}$$

BECAUSE g IS MADE UP OF CONT FUNCS, IT IS CONT EVERYWHERE EXCEPT POSSIBLY AT $X=0$ AND $X=\pi$.

$X=0$

$g(0) = 2$

$\lim_{x\to 0^+} g(x) = 1+\cos 0$

$= 2$

$\Rightarrow$ $\boxed{\text{CONT AT } X=0}$

$X = \pi$

$g(\pi) = 1 + \pi \sin \pi$

$= 1$

$\neq \lim_{x\to\pi^-} g(x)$

$= 1+\cos\pi$

$= 0$

$\Rightarrow$ $\boxed{\text{NOT CONT AT } X=\pi}$

9. (15 points) Differentiate. Use the differentiation rules. Do not simplify.

(a) $\frac{d}{dt}\left(6t^4 + \frac{8}{t^2} - \sqrt[3]{t^2}\right) = \frac{d}{dt}\left(6t^4 + 8t^{-2} - t^{2/3}\right)$

$= \boxed{24t^3 - 16t^{-3} - \frac{2}{3}t^{-1/3}}$

(b) $\frac{d}{dx}\left[5 \cdot \tan x \cdot \sec x\right] = \boxed{5\tan x(\sec x \tan x) + 5(\sec^2 x)(\sec x)}$

(c) $\frac{d}{dx}\left(\frac{x^2\sin x}{x+2}\right) = \boxed{\frac{(x+2)(2x\sin x + x^2\cos x) - (x^2\sin x)(1)}{(x+2)^2}}$

10. (12 points) Let $f(x) = x^2 - 4x + 4$. Use the limit definition of the derivative to find $f'(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^2 - 4(x+\Delta x) + 4\right] - \left[x^2 - 4x + 4\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - 4x - 4\Delta x + 4 - x^2 + 4x - 4}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2 - 4\Delta x}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 4)$$

$$= \boxed{2x - 4}$$

11. (8 points) Let $f(x) = \sqrt{x} + 5x - 2x^2$. Use basic differentiation rules to find $f'(x)$. Then find an equation of the line tangent to the graph of f at $x = 4$.

$$\boxed{f'(x) = \frac{1}{2}x^{-1/2} + 5 - 4x}$$

$$m = f'(4) = \frac{1}{2}(4)^{-1/2} + 5 - 4(4) = \frac{1}{4} + 5 - 16 = -\frac{43}{4}$$

$$x = 4 \Rightarrow y = f(4) = \sqrt{4} + 20 - 2(16) = -10$$

$$\boxed{y + 10 = -\frac{43}{4}(x - 4)}$$