Math 171 - Test 2
October 17, 2013

Name Key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (11 points) The graph of the equation $4y^2 = x^3 + xy^2$ is called a *cissoid.* Find an equation of the line tangent to the cissoid at the point $(2, -2)$.

$$\frac{d}{dx} 4y^2 = \frac{d}{dx}\left(x^3 + xy^2\right)$$

$$8y\frac{dy}{dx} = 3x^2 + y^2 + 2xy\frac{dy}{dx}$$

$$(8y - 2xy)\frac{dy}{dx} = 3x^2 + y^2$$

$$\frac{dy}{dx} = \frac{3x^2+y^2}{8y-2xy}$$

$$\left.\frac{dy}{dx}\right|_{(2,-2)} = \frac{3(2)^2+(-2)^2}{8(-2)-2(2)(-2)} = \frac{16}{-8} = -2$$

TAN LINE AT $(2,-2)$ ...

$$\boxed{y + 2 = -2(x-2)}$$

2. (7 points) Find all critical numbers of the function $f(x) = 3x^{2/5} - 2x^{7/5}$.

$$f'(x) = \frac{6}{5}x^{-3/5} - \frac{14}{5}x^{2/5}$$

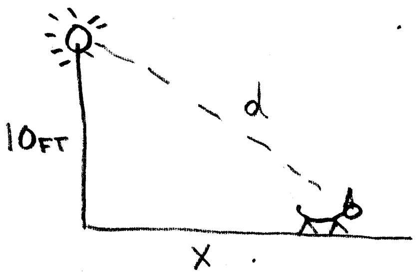
$$= \frac{1}{5}x^{-3/5}(6 - 14x)$$

$$= \frac{6-14x}{5x^{3/5}}$$

$f'(x)$ DNE WHEN $\boxed{x = 0}$

$f'(x) = 0$ WHEN $\boxed{x = \frac{6}{14} = \frac{3}{7}}$

3. (9 points) A dog runs away from a 10-ft lampost at a rate of 4 ft/sec. At what rate is the distance between the light bulb and the dog changing at the moment when the dog is 12 ft from the post?



$\frac{dx}{dt} = 4$. FIND $\frac{dd}{dt}$ WHEN $x = 12$.

$x^2 + 100 = d^2$

$2x \frac{dx}{dt} = 2d \frac{dd}{dt}$

$\therefore \frac{x}{d} \frac{dx}{dt} = \frac{dd}{dt}$

WHEN $x = 12$...

$144 + 100 = d^2 \Rightarrow$

$d = \sqrt{244}$

$$\boxed{\frac{dd}{dt} = \frac{12}{\sqrt{244}}(4) \approx 3.07 \text{ FT/SEC}}$$

4. (6 points) Let $f(x) = x^2 - 3x + 4$. Find a number c that satisfies the conclusion of the Mean Value Theorem for f on $[-1, 2]$.

$$f'(c) = \frac{f(2) - f(-1)}{2 - (-1)}$$

$f'(c) = 2c - 3$

$f(2) = 2$

$f(-1) = 8$

$2c - 3 = \frac{2 - 8}{3} = -2$

$2c = 1$

$$\boxed{c = \frac{1}{2}}$$

5. (12 points) An object is launched upward with an initial speed of 64 ft/sec over the side of a 192-ft cliff. (Use $g = 32\,\text{ft/sec}^2$.)

(a) Determine the function that gives the object's height at time t.

$$S(t) = -16t^2 + 64t + 192$$

(b) What is the object's velocity after 4 seconds?

$$S'(t) = -32t + 64$$

$$S'(4) = \boxed{-64\ \text{FT/SEC}}$$

(c) What is the object's maximum height?

$$S'(t) = 0 \Rightarrow t = 2$$

$$S(2) = -16(4) + 64(2) + 192$$
$$= \boxed{256\ \text{FT}}$$

(d) When will the object hit the ground?

$$S(t) = 0 \Rightarrow -16t^2 + 64t + 192 = 0$$

$$-16(t^2 - 4t - 12) = 0$$
$$-16(t-6)(t+2) = 0 \Rightarrow t = \boxed{6\ \text{SEC}}$$

(e) Determine the object's acceleration function.

$$S''(t) = \boxed{-32} \quad \text{IN FT/SEC}^2$$

6. (12 points) Differentiate. Do not simplify.

(a) $\frac{d}{dw}\sqrt[3]{w^2+w} = \frac{d}{dw}\left(w^2+w\right)^{1/3}$

$= \frac{1}{3}\left(w^2+w\right)^{-2/3}(2w+1)$

(b) $\frac{d}{dt}\left(\frac{t+5}{t^2+5}\right)^2 = 2\left(\frac{t+5}{t^2+5}\right)\left(\frac{(t^2+5)(1)-(t+5)(2t)}{(t^2+5)^2}\right)$

(c) $\frac{d}{dx}\cos^2(6x) = 2\cos(6x)\frac{d}{dx}\cos(6x)$

$= 2\cos(6x)(-\sin 6x)(6)$

7. (6 points) Suppose f and g are increasing, differentiable functions. Is $f+g$ an increasing function? Explain your reasoning. Is $f-g$ an increasing function?

$f+g$? $\boxed{\text{YES}}$ IF $f'(x) > 0$ AND $g'(x) > 0$, THEN

$f'(x)+g'(x) > 0$.

$f-g$? $\boxed{\text{NOT NECESSARILY}}$

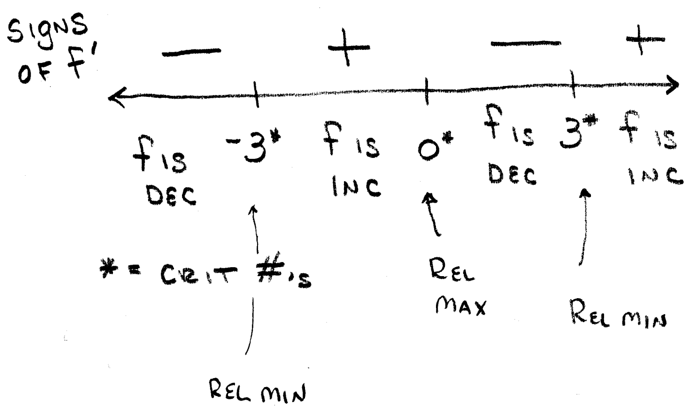
FOR EXAMPLE $f(x) = x$, $g(x) = 2x$ ARE INCREASING.

$f(x) - g(x) = -x$, DECREASING!

8. (12 points) Consider the function $f(x) = x^4 - 18x^2 + 9$. Determine open intervals on which f is increasing/decreasing. Also identify all relative extreme values.

$$f'(x) = 4x^3 - 36x$$
$$= 4x(x^2-9) = 4x(x-3)(x+3)$$

$$f'(x) = 0 \Rightarrow x = 0, \quad x = 3, \quad x = -3$$



$f(-3) = -72$ IS A REL MIN

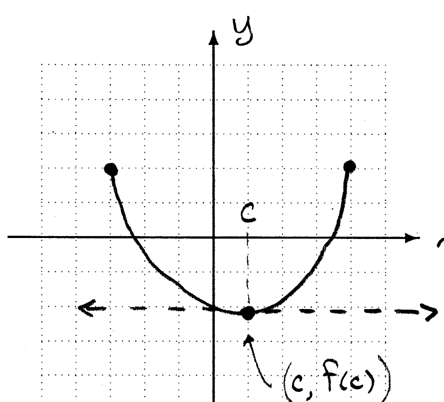
$f(0) = 9$ IS A REL MAX

$f(3) = -72$ IS A REL MIN

f IS DECREASING ON $(-\infty, -3) \cup (0, 3)$

f IS INCREASING ON $(-3, 0) \cup (3, \infty)$

9. (6 points) Sketch the graph of a nonconstant function f that is continuous on $[-3, 4]$, differentiable on $(-3, 4)$, and that satisfies $f(-3) = f(4) = 2$. Then identify a point on the graph that satisfies the conclusion of Rolle's Theorem.



THIS IS SUCH A POINT!

$f'(c) = 0$

ACCORDING TO ROLLE'S THM, THERE IS SUCH A POINT.

10. (6 points) Find $g''(x)$ if $g(x) = (x^3 + 4x)^5$.

$$g'(x) = 5(x^3+4x)^4(3x^2+4)$$

$$\boxed{g''(x) = 20(x^3+4x)^3(3x^2+4)^2 + 5(x^3+4x)^4(6x)}$$

11. (8 points) Find the absolute maximum and minimum values of $g(x) = x + 2\cos x$ on the interval $[0, 2]$.

$g'(x) = 1 - 2\sin x = 0$

$\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{6}$

$g(0) = 2$

$g\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \sqrt{3} \approx 2.25565$

$g(2) = 2 + 2\cos 2 \approx 1.16771$

Abs min is $g(2) \approx 1.16771$

Abs max is $g\left(\frac{\pi}{6}\right) \approx 2.25565$

12. (5 points) Determine the higher-order derivative: $\frac{d^3}{dx^3}(2x^8 - 5x^4 - 5\sin x)$

$$= \frac{d^2}{dx^2}(16x^7 - 20x^3 - 5\cos x)$$

$$= \frac{d}{dx}(112x^6 - 60x^2 + 5\sin x)$$

$$= \boxed{672x^5 - 120x + 5\cos x}$$