Math 171 - Test 3a

November 14, 2013

Name Key

Score ________

Show all work. Supply explanations when necessary.

1. (6 points) Let $g(x) = x^4 + \cos(20x)$. Without looking at the graph of g, determine whether the graph is concave up or concave down at the point where $x = 0.7$.

$g'(x) = 4x^3 - 20 \sin 20x$

$g''(x) = 12x^2 - 400 \cos 20x$

$g''(0.7) = -48.814887\ldots < 0$

$\Rightarrow$ $\boxed{\text{GRAPH IS CONCAVE DOWN}}$

2. (4 points) Find the horizontal asymptote(s) of the graph of R.

$$R(x) = \frac{4x^6 + 7x^2 + 1}{2x^7 - 8x^6 + 7x^3 + 5}$$

R IS A RATIONAL FUNCTION WITH DEGREE OF NUMERATOR LESS THAN DEGREE OF DENOMINATOR.

$\boxed{\text{THE ONLY HA IS } y = 0.}$

3. (6 points) Find the function f such that $f'(x) = 10x^4 + \dfrac{1}{x^2}$ and $f(1) = 4$.

$f(x) = \int (10x^4 + x^{-2})\, dx$

$= 2x^5 - \frac{1}{x} + C$

$f(1) = 4 = 2(1)^5 - \frac{1}{1} + C \Rightarrow C = 3$

$\boxed{f(x) = 2x^5 - \frac{1}{x} + 3}$

4. (6 points) Use the linearization of $h(x) = \sqrt[3]{x} + \sqrt[5]{x}$ at $x = 1$ to approximate $\sqrt[3]{1.1} + \sqrt[5]{1.1}$.

$L(x) = h(c) + h'(c)(x-c)$

$c = 1$

$h(1) = 2$

$h'(x) = \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5}$

$h'(1) = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

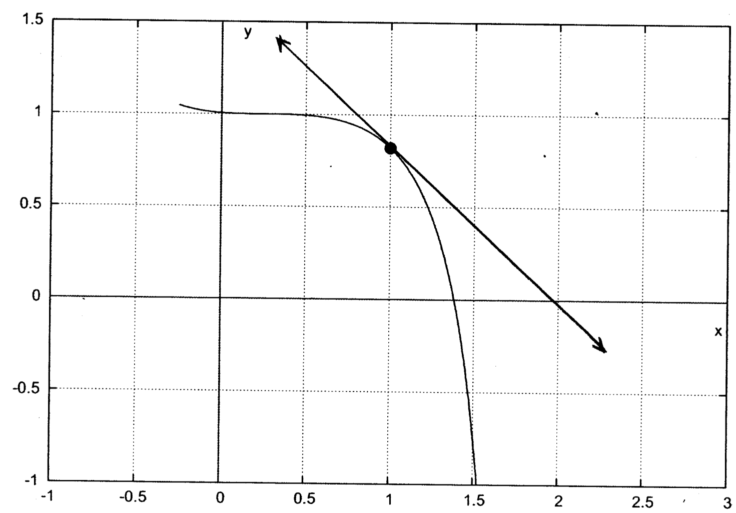
$L(x) = 2 + \frac{8}{15}(x-1)$

$f(1.1) \approx L(1.1)$

$\approx \boxed{2.053}$

5. (4 points) The graph of $y = f(x)$ is shown below. Suppose you use Newton's method, starting with $x_0 = 1$, to approximate a solution of $f(x) = 0$. Which one of the following numbers would be closest to x_1? Explain your reasoning.

(a) 1.9
(b) 1.35
(c) 2.5
(d) 1.0



THE LINEARIZATION OF f AT $X = 1$ CROSSES THE X-AXIS AT A POINT CLOSE TO $X = 1.9$.

THEREFORE $X_0 = 1 \Rightarrow X_1 \approx 1.9$.

6. (4 points) Evaluate the limit: $\lim_{x\to-\infty} \dfrac{2x^2+7x-3}{5x^2-x}$

$$\lim_{x\to-\infty} \frac{2x^2+7x-3}{5x^2-x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to-\infty} \frac{2+\frac{7}{x}-\frac{3}{x^2}}{5-\frac{1}{x}}$$

$$= \frac{2+0-0}{5-0}$$

$$= \boxed{\frac{2}{5}}$$

7. (5 points) Use Newton's method, starting with $x_0 = -2$, to approximate the solution of the equation $x+1 = \sin 2x$. Which one of these numbers is closest to your value of x_2?

(a) -1.385
(b) -1.377
(c) -1.720
(d) 0.957

$f(x) = x+1-\sin 2x$

$f'(x) = 1-2\cos 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_0 = -2$

$x_1 = -1.238585269\ldots$

$x_2 = -1.385414626\ldots$

$x_3 = -1.377353615\ldots$

$x_4 = -1.377336877\ldots$

8. (6 points) Let $y = f(x) = 2\sin x + 5\cos x$. Use differentials to approximate $\Delta y = f(3) - f(\pi)$.

$dy = (2\cos x - 5\sin x)\,dx$

$\Delta y \approx (2\cos x - 5\sin x)\,\Delta x$

$\Delta x = 3-\pi \approx -0.14159 \Rightarrow \Delta y \approx (2\cos\pi - 5\sin\pi)(-0.14159)$

$\approx \boxed{0.2832}$

9. (5 points) Evaluate the limit: $\displaystyle\lim_{x\to-\infty} \frac{3x}{\sqrt{2x^2+5}}$.

$$\lim_{x\to-\infty} \frac{3x}{\sqrt{2x^2+5}} \cdot \underbrace{\frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}} = \lim_{x\to-\infty} \frac{-3}{\sqrt{2+\frac{5}{x^2}}}}_{\text{SINCE } x<0,\ \sqrt{x^2}=-x} = \frac{-3}{\sqrt{2+0}} = \boxed{\frac{-3}{\sqrt{2}}}$$

10. (8 points) Let $g(x) = \dfrac{x+3}{\sqrt{x}}$. Find open intervals on which the graph of g is concave up/down. Identify all inflection points.

$$g(x) = x^{1/2} + 3x^{-1/2}$$

$$g'(x) = \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{-3/2}$$

$$g''(x) = -\frac{1}{4}x^{-3/2} + \frac{9}{4}x^{-5/2}$$

$$g''(x) = 0 \Rightarrow \underbrace{\frac{9}{4}x^{-5/2} = \frac{1}{4}x^{-3/2}}_{\text{MULT BY } 4x^{5/2} \text{ TO GET}} \quad 9 = x$$

$g''(x)$ DNE WHEN $x=0$

BUT 0 IS NOT IN THE DOMAIN OF g.

SIGNS OF g''

$g''(4) \approx 0.04$ $\quad g''(10) \approx -0.0008$

$+$ $\quad$ $-$

0 $\quad$ CU $\quad$ 9 $\quad$ CD

$$\boxed{\begin{array}{l}\text{GRAPH IS CU ON } (0,9) \\ \text{GRAPH IS CD ON } (9,\infty) \\ \text{INFLECTION PT IS } (9,4)\end{array}}$$

11. (8 points) Evaluate each indefinite integral.

(a) $\int \left(\sqrt[7]{x^5} + 6x^2 - \frac{8}{x^3} \right) dx$

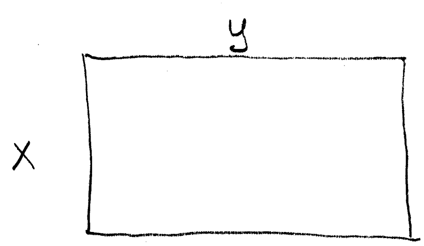
$$= \int \left(x^{5/7} + 6x^2 - 8x^{-3} \right) dx$$

$$= \boxed{\frac{7}{12} x^{12/7} + 2x^3 + 4x^{-2} + C}$$

(b) $\int (5\sec^2 t + \cos t)\, dt$

$$= \boxed{5\tan t + \sin t + C}$$

12. (8 points) The length and width of a rectangle are reciprocals of one another. Find the minimum perimeter of such a rectangle.



MINIMIZE $P = 2x + 2y$

s.t. $y = \frac{1}{x}$

$$P(x) = 2x + \frac{2}{x}, \quad x > 0$$

$$P'(x) = 2 - \frac{2}{x^2} = 0 \Rightarrow x = 1$$

AND $y = 1$

$$P''(x) = \frac{4}{x^3} \Rightarrow P''(1) > 0$$

$\Rightarrow$ $x = 1$ GIVES A MIN.

MINIMUM PERIMETER IS 4 WHEN LENGTH = WIDTH = 1

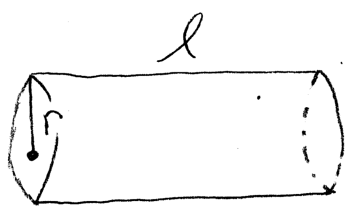
Math 171 - Test 3b
November 14, 2013

Name key

Score ______

Show all work. Supply explanations when necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

1. (12 points) A package to be sent by a postal service can have a maximum combined length and girth (perimeter of a cross section) of 108 in. Find the dimensions of the cylindrical package of maximum volume that can be sent.



MAXIMIZE $V = \pi r^2 \ell$

SUBJECT TO

$\ell + 2\pi r = 108$

$\ell = 108 - 2\pi r$

$V(r) = \pi r^2 (108 - 2\pi r)$

$V(r) = 108\pi r^2 - 2\pi^2 r^3, \quad 0 \le r \le \frac{108}{2\pi}$

$V'(r) = 216\pi r - 6\pi^2 r^2$

$V'(r) = 0 \Rightarrow 2\pi r(108 - 3\pi r) = 0$

$\Rightarrow r = 0$ or $r = \frac{108}{3\pi}$

CHECKING CRITICAL #'S AND ENDPTS...

$V(0) = 0$

$V\left(\frac{108}{3\pi}\right) \approx 14,851$ ← ABS MAX!

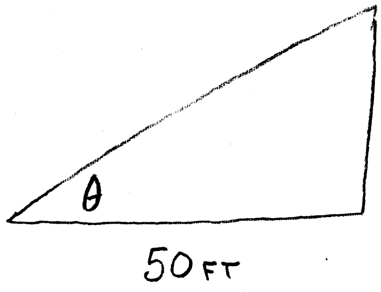
$V\left(\frac{108}{2\pi}\right) = 0$

DIMENSIONS ARE

$r = \frac{108}{3\pi}$ in

$\ell = 36$ in

2. (8 points) A surveyor standing exactly 50 feet from the base of a large tree measures the angle of elevation to the top of the tree as 1.25 radians. Use differentials to determine how accurately the angle must be measured if the percent error in estimating the height of the tree is to less than 6%.



TREE OF HEIGHT h

$$h = 50 \tan \theta$$

$$dh = 50 \sec^2\theta \, d\theta$$

$$\Downarrow$$

$$\Delta h \approx 50 \sec^2\theta \, \Delta\theta$$

$\theta = 1.25$. FIND $\Delta\theta$

SO THAT $\dfrac{\Delta h}{h} = 0.06$

$$\frac{\cancel{50} \sec^2(1.25)\,\Delta\theta}{\cancel{50} \tan(1.25)} = 0.06$$

$$\Delta\theta = \frac{0.06 \tan(1.25)}{\sec^2(1.25)}$$

$$= 0.06 \tan(1.25) \cos^2(1.25)$$

$$= 0.06 \sin(1.25) \cos(1.25)$$

$$= 0.017954\ldots$$

$$\boxed{\Delta\theta \approx \pm 0.018 \text{ RADIANS}}$$

3. (10 points) Use Newton's method to find good approximations for the three zeros of $p(x) = x^3 - 5x^2 + x + 1$.

SOLVE

$p(x) = 0.$

$p(x) = x^3 - 5x^2 + x + 1$

$p'(x) = 3x^2 - 10x + 1$

$$X_{N+1} = X_N - \frac{p(X_N)}{p'(X_N)}$$

$X_0 = -0.5$

$X_1 = -0.37037...$

$X_2 = -0.34944...$

$X_3 = -0.34889...$

$\boxed{X_4 = -0.3488942175...}$

X_5 = SAME AS X_4

$X_0 = 0.5$

$X_1 = 0.61538...$

$X_2 = 0.60416...$

$X_3 = 0.604068...$

$\boxed{X_4 = 0.6040681398...}$

X_5 = SAME AS X_4

$X_0 = 5.5$

$X_1 = 4.91156...$

$X_2 = 4.75579...$

$X_3 = 4.744878...$

$X_4 = 4.74482...$

$\boxed{X_5 = 4.744826078...}$

X_6 = SAME AS X_5