

**Math 171 - Quiz 6**

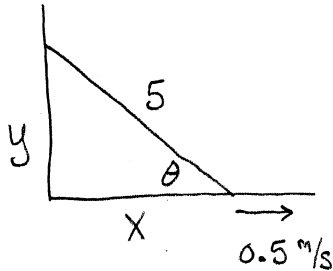
October 2, 2014

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) A 5-m-long ladder is leaning against a wall. If the lower end of the ladder slides away from the wall at the rate of 0.5 m/s, at what rate is the angle of inclination,  $\theta$ , of the ladder with respect to the ground changing when the lower end of the ladder is 4 m from the wall?



$$\cos \theta = \frac{x}{5} \Rightarrow -\sin \theta \frac{d\theta}{dt} = \frac{1}{5} \frac{dx}{dt}$$

When  $x=4$ ,

$$y = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$$

$$\sin \theta = \frac{3}{5}$$

$$\frac{dx}{dt} = 0.5$$

Find  $\frac{d\theta}{dt}$  when  $x=4$

$$\frac{d\theta}{dt} = -\frac{1}{5 \sin \theta} \frac{dx}{dt} = -\frac{0.5}{3}$$

$$\approx -0.17 \text{ rad/s}$$

2. (5 points) The graph of  $(x^2 + y^2)^2 = x^2 - y^2$  is called a *lemniscate*. Find  $dy/dx$ .

$$\frac{d}{dx} (x^2 + y^2)^2 = \frac{d}{dx} (x^2 - y^2)$$

$$2(x^2 + y^2) \left( 2x + 2y \frac{dy}{dx} \right) = 2x - 2y \frac{dy}{dx}$$

$$4x(x^2 + y^2) + 4y(x^2 + y^2) \frac{dy}{dx} = 2x - 2y \frac{dy}{dx}$$

$$4y(x^2 + y^2) \frac{dy}{dx} + 2y \frac{dy}{dx} = 2x - 4x(x^2 + y^2)$$

$$\frac{dy}{dx} = \frac{2x - 4x(x^2 + y^2)}{4y(x^2 + y^2) + 2y}$$