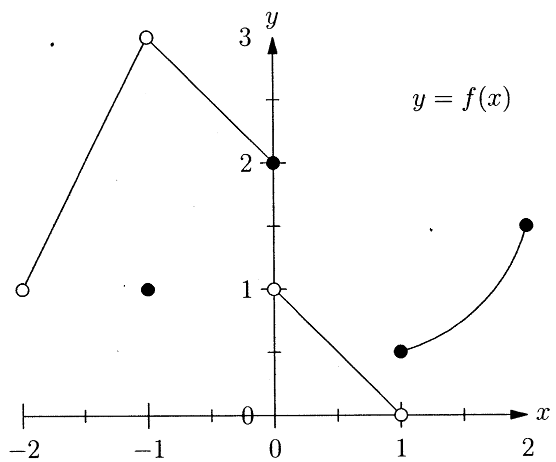
Math 171 - Test 1
September 11, 2014

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. When evaluating limits, you may need to use $+\infty$, $-\infty$, or DNE (does not exist).

1. (10 points) Referring to the graph shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x\to -1} f(x) = 3$

(b) $\lim_{x\to 0^-} f(x) = 2$

(c) $f(1) = 0.5$

(d) $\lim_{x\to 0^+} f(x) = 1$

(e) $\lim_{x\to -2} f(x)$ DNE because f is not defined on an interval around $x=-2$.

However, $\lim_{x\to -2^+} f(x) = 1$

2. (9 points) Determine whether each statement is true (T) or false (F).

(a) __F__ If f is defined at $x = c$, then $\lim_{x \to c} f(x)$ exists.

(b) __F__ $\lim_{x \to 0} \sqrt{x} = 0$ BUT $\lim_{X \to 0^+} \sqrt{X} = 0$

(c) __F__ If $h(5) = 0$, then $\lim_{x \to 5} h(x) = 0$. ONLY IF h IS CONT. AT $X = 5$

(d) __T__ If $\lim_{x \to 1} f(x) = f(1)$, then f is continuous at $x = 1$.

(e) __F__ If $\lim_{t \to 0^+} g(t) = 2$, then $\lim_{t \to 0} g(t) = 2$. BUT TRUE THE OTHER WAY AROUND

(f) __F__ If f and g are polynomials and $g(6) = 0$, then the graph of $\dfrac{f(x)}{g(x)}$ must have a vertical asymptote at $x = 6$. FOR EXAMPLE, $\dfrac{X-6}{X-6}$ HAS NO V.A.

3. (6 points) Use a table of numerical values to approximate the following limit. Your table must show function values at four or more points.

$$\lim_{x \to 0^+} \frac{5^x - 1}{3x}$$

X	$\dfrac{5^X - 1}{3X}$
0.1	0.58206
0.01	0.54082
0.001	0.53691
0.0001	0.53652

IT LOOKS LIKE $\lim_{X \to 0^+} \dfrac{5^X - 1}{3X} \approx 0.54$

4. (6 points) Is g continuous everywhere? Carefully explain your reasoning.

$$g(x) = \begin{cases} x^2 + 2, & x \le 0 \\ 1 + \cos x, & 0 < x < \pi \\ 1 + x \sin x, & x \ge \pi \end{cases}$$

g IS DISCONTINUOUS ONLY AT $X = \pi$

BECAUSE EACH PIECE IS CONTINUOUS EVERYWHERE, WE NEED ONLY CHECK IF THE PIECES FIT TOGETHER. WE LOOK AT $X = 0$ & $X = \pi$.

$X = 0$: $\lim_{X \to 0^-} g(x) = \lim_{X \to 0} (X^2 + 2) = 2$

$\lim_{X \to 0^+} g(x) = \lim_{X \to 0} (1 + \cos X) = 2$

CONT AT $X = 2$

$X = \pi$: $\lim_{X \to \pi^-} g(x) = \lim_{X \to \pi} (1 + \cos X) = 0$

$\lim_{X \to \pi^+} g(x) = \lim_{X \to \pi} (1 + X \sin X) = 1$

DISCONT AT $X = \pi$

5. (10 points) The table below gives values of the **continuous** functions f and g at selected points.

x	-2	-1	0	1	2
$f(x)$	1.5	2.75	2	-1.75	-3.4
$g(x)$	4	2.1	1.3	0	1.75

(a) Find $\lim_{x \to -2^+} g(x)$.

BECAUSE g IS CONTINUOUS, $\lim_{x \to -2^+} g(x) = g(-2) = \boxed{4}$

(b) Find $\lim_{x \to 0} g(f(x))$.

$= g(f(0)) = g(2) = \boxed{1.75}$

(c) Find $f(3)$ if $\lim_{x \to 3} f(x) = 5.25$.

$\lim_{x \to 3} f(x) = f(3)$ BECAUSE f IS CONT. $\boxed{f(3) = 5.25}$

(d) Find the smallest interval on which you can be sure that the equation $g(x) = 2.25$ has at least one solution. Explain your reasoning.

$g(-2) = 4$

$g(-1) = 2.1$ $\Longrightarrow$ $\boxed{g(x) = 2.25 \text{ SOMEWHERE ON } (-2,-1)}$

(e) Assuming $g(x) \geq 0$ for all x, what can be said about $\lim_{x \to 1} \frac{f(x)}{g(x)}$?

$\lim_{x \to 1} \frac{f(x)}{g(x)}$ HAS FORM $\frac{-1.75}{0}$ SINCE, $g(x) \geq 0$, $\lim_{x \to 1} \frac{f(x)}{g(x)} = \boxed{-\infty}$

(f) (Bonus: 2 points) What is the name of the theorem you used in part (d)?

INTERMEDIATE VALUE THEOREM

6. (24 points) Determine each limit analytically, or explain why the limit does not exist. You may need to use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x\to 0} \frac{(x-2)^2-4}{x}$ $\frac{0}{0}$ MORE WORK

$$= \lim_{x\to 0} \frac{x^2-4x+4-4}{x} = \lim_{x\to 0} \frac{x^2-4x}{x}$$

$$= \lim_{x\to 0} \frac{\cancel{x}(x-4)}{\cancel{x}} = \lim_{x\to 0} (x-4) = \boxed{-4}$$

(b) $\lim_{r\to 0^+} \frac{\sqrt{2+r}-\sqrt{2}}{r}$ $\frac{0}{0}$ MORE WORK

$$\cdot \frac{\sqrt{2+r}+\sqrt{2}}{\sqrt{2+r}+\sqrt{2}} = \lim_{x\to 0^+} \frac{\cancel{2}+r-\cancel{2}}{r(\sqrt{2+r}+\sqrt{2})}$$

$$= \lim_{r\to 0^+} \frac{1}{\sqrt{2+r}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$$

(c) $\lim_{x\to 4^-} \left(\frac{x-4}{x^2-8x+16}\right) = \lim_{x\to 4^-} \frac{x-4}{(x-4)^2} = \lim_{x\to 4^-} \frac{1}{x-4}$ $\frac{1}{0}$ INF LIMIT

TO THE LEFT OF $x=4$

$$\frac{1}{x-4} = \frac{+}{-} = - \qquad \lim_{x\to 4^-} \frac{1}{x-4} = \boxed{-\infty}$$

(d) $\lim_{z\to 6} \frac{(z-4)^2+2(z+1)}{z+3} = \frac{(6-4)^2+2(6+1)}{6+3} = \frac{4+14}{9} = \frac{18}{9} = \boxed{2}$

Formal Definition of Limit: The statement $\lim_{x \to c} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that $0 < |x - c| < \delta$ implies that $|f(x) - L| < \epsilon$.

7. (6 points) Compute $\lim_{x \to -2} (2x - 2)$ and then, referring to the formal definition of limit, find a δ that corresponds to $\epsilon = 0.001$.

$$\lim_{x \to -2} (2x - 2) = -6$$

$$|2x - 2 + 6| < \epsilon$$

$$\Updownarrow$$

$$|2x + 4| < \epsilon$$

$$\Updownarrow$$

$$2|x + 2| < \epsilon$$

$$\Longleftrightarrow \quad |x + 2| < \frac{\epsilon}{2}$$

IF $\epsilon = 0.001$, THEN $\delta = 0.0005$

8. (5 points) Given that $-x^2 \le x^2 \cos\frac{1}{x} \le x^2$ for $x \neq 0$, compute $\lim_{x \to 0} x^2 \cos\frac{1}{x}$. Explain.

$$\lim_{x \to 0} (-x^2) \le \lim_{x \to 0} x^2 \cos\frac{1}{x} \le \lim_{x \to 0} x^2$$

$$0 \le \lim_{x \to 0} x^2 \cos\frac{1}{x} \le 0$$

$$\Rightarrow \lim_{x \to 0} x^2 \cos\frac{1}{x} = 0$$ By Squeeze Thm.

9. (6 points) Use the fact that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ to compute $\lim_{x \to 0} \left[\frac{4 \tan x \cos x}{x(1 + \sec x)}\right]$.

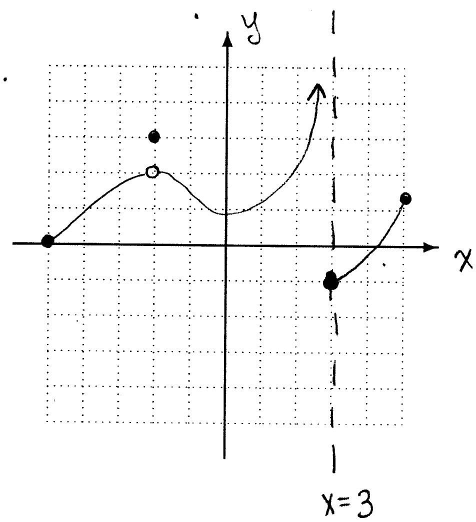
$$\lim_{x \to 0} \frac{4 \tan x \cos x}{x(1 + \sec x)} = \lim_{x \to 0} \frac{4 \sin x}{x(1 + \sec x)}$$

$$= \lim_{x \to 0} \left[\frac{\sin x}{x} \cdot \frac{4}{1 + \sec x}\right] = \left(\lim_{x \to 0} \frac{\sin x}{x}\right)\left(\lim_{x \to 0} \frac{4}{1 + \sec x}\right)$$

$$= (1)\left(\frac{4}{2}\right) = \boxed{2}$$

10. (6 points) Sketch the graph of a function f such that

- f is defined for all real numbers between -5 and 5,
- $f(-2) = 3$,
- f has a removable discontinuity at $x = -2$,
- $\lim_{x \to 3^-} f(x) = \infty$, and
- $\lim_{x \to 3^+} f(x) = -1$.



11. (6 points) Find and classify the discontinuites of $R(x) = \dfrac{x^2 + 2x - 8}{x^2 - x - 2}$ $= \dfrac{(X+4)(X-2)}{(X+1)(X-2)}$

R IS DISCONTINUOUS ONLY AT $X = -1$ AND $X = 2$

THIS DISCONT AT $X = 2$ IS REMOVABLE BECAUSE $\lim_{X \to 2} R(x) = \lim_{X \to 2} \dfrac{X+4}{X+1} = \dfrac{6}{3}$

THIS DISCONT AT $X = -1$ IS INFINITE BECAUSE SUBS INTO R GIVES FORM $\dfrac{k \neq 0}{0}$.

12. (6 points) Explain why $\lim_{x \to 1} \dfrac{x^2 - x}{|x - 1|}$ fails to exist. Show work to justify your answer.

$$|X-1| = \begin{cases} -(X-1), & X < 1 \\ X-1, & X \geq 1 \end{cases}$$

$$\lim_{X \to 1^-} \frac{X^2 - X}{|X-1|} = \lim_{X \to 1^-} \frac{X\cancel{(X-1)}}{-\cancel{(X-1)}} = -1$$

$$\lim_{X \to 1^+} \frac{X^2 - X}{|X-1|} = \lim_{X \to 1^+} \frac{X\cancel{(X-1)}}{\cancel{(X-1)}} = 1$$

LEFT LIMIT $\neq$ RIGHT LIMIT