Math 171 - Test 2
October 9, 2014

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. Suppose you throw a pebble upwards with a velocity of 4 ft/sec from the top of a 300-ft building. Use the position function $s(t) = -16t^2 + v_0 t + s_0$, where s represents height (in feet) at time t (in seconds), to solve the following problems.

   (a) (2 points) Determine the function that gives the pebble's height at time t.

   $$\boxed{s(t) = -16t^2 + 4t + 300}$$

   (b) (2 points) Determine the function that gives the pebble's velocity at time t.

   $$\boxed{v(t) = s'(t) = -32t + 4}$$

   (c) (2 points) Determine the pebble's velocity after 3 seconds. (Include units with your answer.)

   $$v(3) = -32(3) + 4 = -92$$

   $$\boxed{-92\ \text{FT}/\text{s}}$$

   (d) (2 points) What is the acceleration of the pebble? (Include units with your answer.)

   $$a(t) = v'(t) = -32$$

   $$\boxed{-32\ \text{FT}/\text{s}^2}$$

   (e) (4 points) Determine the pebble's maximum height. (Include units with your answer.)

   $$v(t) = 0 \Rightarrow -32t + 4 = 0 \Rightarrow t = \frac{1}{8}$$

   $$s\left(\tfrac{1}{8}\right) = -16\left(\tfrac{1}{8}\right)^2 + 4\left(\tfrac{1}{8}\right) + 300$$

   $$= 300.25$$

   $$\boxed{300.25\ \text{FT}}$$

2. (20 points) Differentiate. Do not simplify.

(a) $\frac{d}{dr}\left(7r^5 + 4r - 8\sqrt{r} + \frac{17}{r^2}\right) = \frac{d}{dr}\left(7r^5 + 4r - 8r^{1/2} + 17r^{-2}\right)$

$= \boxed{35r^4 + 4 - 4r^{-1/2} - 34r^{-3}}$

(b) $\frac{d}{dx}\left[3x^2 \sec x\right] = \boxed{6x \sec x + 3x^2 \sec x \tan x}$

(c) $\frac{d}{dx}\left[(2x-4)^3(4x-1)^5\right]$

$= \boxed{3(2x-4)^2(2)(4x-1)^5 + (2x-4)^3(5)(4x-1)^4(4)}$

(d) $\frac{d}{dt}\sin(2\pi t^2) = \boxed{\cos(2\pi t^2)(4\pi t)}$

3. (4 points) Which one of the following best describes the line tangent to the graph of $f(x) = (3x - 12)^{1/3}$ at the point $(4, 0)$? (Briefly explain, or show work, to receive full credit.)

   (a) The tangent line is horizontal.

   (b) The tangent line is vertical.

   (c) A unique tangent line does not exist.

   (d) The tangent line cannot be determined from the given information.

$$f'(x) = \frac{1}{3}(3x-12)^{-2/3}(3) = \frac{1}{\sqrt[3]{(3x-12)^2}}$$

$f'(4)$ DNE BECAUSE OF $\frac{1}{0}$

4. (12 points) The table below gives the values of the functions f and g and their derivatives at selected values of x.

x	-2	-1	2
$f(x)$	1	3	-2
$f'(x)$	2	-1	-1
$g(x)$	2	0	-2
$g'(x)$	-3	-2	1

(a) If $h(x) = f(x) \cdot g(x)$, compute $h'(-1)$.

$$h'(x) = f(x)g'(x) + f'(x)g(x)$$

$$h'(-1) = f(-1)g'(-1) + f'(-1)g(-1) = (3)(-2) + (-1)(0)$$

$$= \boxed{-6}$$

(b) If $h(x) = f(g(x))$, compute $h'(2)$.

$$h'(x) = f'(g(x))\,g'(x)$$

$$h'(2) = f'(g(2))\,g'(2) = f'(-2)\,(1) = 2\,(1) = \boxed{2}$$

(c) If $h(x) = \dfrac{3f(x)}{g(x)}$, compute $h'(2)$.

$$h'(x) = \frac{3g(x)f'(x) - 3f(x)g'(x)}{[g(x)]^2}$$

$$h'(2) = \frac{3g(2)f'(2) - 3f(2)g'(2)}{[g(2)]^2} = \frac{3(-2)(-1) - 3(-2)(1)}{(-2)^2}$$

$$= \frac{12}{4} = \boxed{3}$$

5. (8 points) Find the slope of the line tangent to the graph of $x^2y + y^3 = 2x^2$ at the point $(-1, 1)$.

$$\frac{d}{dx}\left(x^2y + y^3\right) = \frac{d}{dx}\left(2x^2\right)$$

$$x^2\frac{dy}{dx} + 2xy + 3y^2\frac{dy}{dx} = 4x$$

$$\left(x^2 + 3y^2\right)\frac{dy}{dx} = 4x - 2xy$$

$$\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + 3y^2}$$

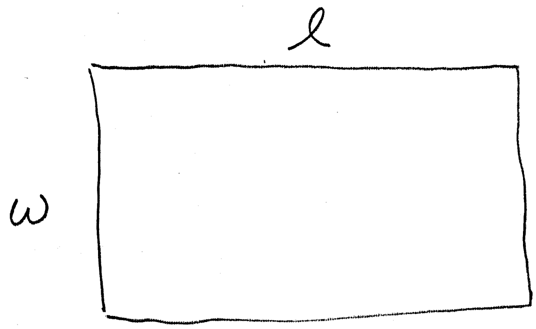
$$\left.\frac{dy}{dx}\right|_{(-1,1)} = \frac{-4+2}{1+3} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$$

6. (6 points) Use the quotient rule to derive the formula for the derivative of $y = \cot x$.

$$\frac{d}{dx}\frac{\cos x}{\sin x} = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$$

$$= \frac{-\left(\sin^2 x + \cos^2 x\right)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

7. (10 points) Consider a rectangle whose length is increasing at 2 centimeters per second and whose width is decreasing at 3 centimeters per second. Find the rate at which the area is changing at the moment the length is 20 centimeters and the width is 24 centimeters.



$$\frac{d\ell}{dt} = 2, \quad \frac{dw}{dt} = -3$$

FIND $\frac{dA}{dt}$ WHEN $\ell = 20$, $w = 24$

$$A = \ell w$$

$$\frac{dA}{dt} = \ell\frac{dw}{dt} + \frac{d\ell}{dt}w$$

WHEN $\ell = 20$, $w = 24$

$$\frac{dA}{dt} = 20(-3) + (2)(24)$$

$$= \boxed{-12 \ \mathrm{cm}^2/\mathrm{s}}$$

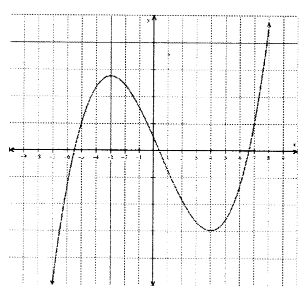
8. (10 points) Let $f(x) = x^2 + 5x$. Use the **limit definition of the derivative** to find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left[(x+h)^2 + 5(x+h)\right] - \left[x^2 + 5x\right]}{h}$$

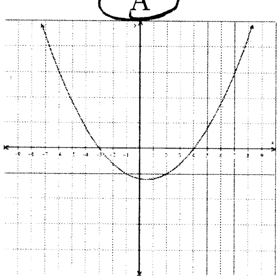
$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + 5x + 5h - x^2 - 5x}{h} = \lim_{h \to 0} \frac{h(2x + h + 5)}{h}$$

$$= \lim_{h \to 0} (2x + h + 5) = 2x + 5$$

9. (5 points) The graph of $g(x)$ is shown below. Choose the lettered graph that best represents the graph of $g'(x)$. Provide a one or two sentence explanation.



A



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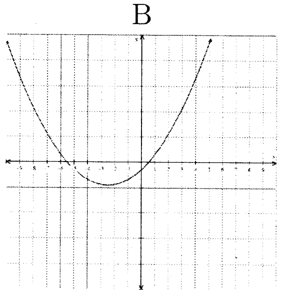

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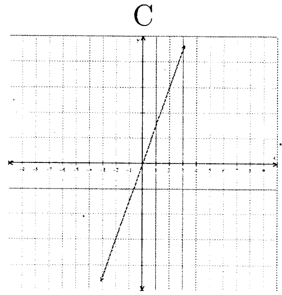

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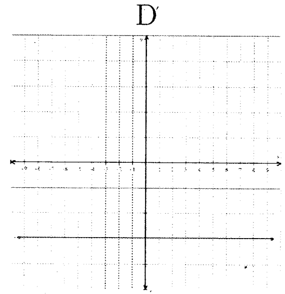

DERIVATIVE IS POSITIVE ON $(-\infty, -3)$,
NEGATIVE ON $(-3, 4)$,
POSITIVE ON $(4, \infty)$,
AND ZERO AT -3 & 4.

10. (8 points) Let $g(x) = 3x^4 - 4x^3 - 72x^2$ on $[-5, 2]$.

(a) Find the critical numbers of g.

$g'(x) = 12x^3 - 12x^2 - 144x$

$= 12x(x^2 - x - 12) = 12x(x-4)(x+3)$

$g'(x) = 0 \Rightarrow x = 0,\ x = 4,\ x = -3$ (X=4 IS NOT IN $[-5,2]$)

$g'(x)$ DNE NOWHERE

CRITICAL #'S ARE $\boxed{x = 0 \text{ \& } x = -3}$

(b) Find the absolute extrema.

CRIT #'S

$g(0) = 0$

$g(-3) = \boxed{-297 \leftarrow \text{ABS MIN}}$

ENDPTS

$g(-5) = \boxed{575 \leftarrow \text{ABS MAX}}$

$g(2) = -272$

11. (5 points) Determine the higher-order derivative: $\dfrac{d^3}{dx^3}\underbrace{(2x^8 - 5x^4 - 5\sin x)}_{f(x)}$

$f'(x) = 16x^7 - 20x^3 - 5\cos x$

$f''(x) = 112x^6 - 60x^2 + 5\sin x$

$\boxed{f'''(x) = 672x^5 - 120x + 5\cos x}$