Math 171 - Test 3a

November 13, 2014

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Find the absolute extreme values of $f(x) = (3x - 9)^{2/3}$ over the interval $[1, 6]$.

$$f'(x) = \frac{2}{3}(3x-9)^{-1/3}(3) = 2(3x-9)^{-1/3} = \frac{2}{\sqrt[3]{3x-9}}$$

$f'(x) = 0$ NEVER

$f'(x)$ DNE WHEN $3x-9=0$

$x = 3$

CRIT # IS $x = 3$

ENDPTS ARE $x = 1$, $x = 6$

$$f(1) = (-6)^{2/3} \approx 3.3$$

$$\boxed{\begin{array}{l} f(6) = (9)^{2/3} \approx 4.3 \leftarrow \text{ABS MAX} \\ f(3) = 0 \leftarrow \text{ABS MIN} \end{array}}$$

2. (8 points) Use Newton's method to approximate the unique solution of the equation $\sin(4x) = 75 - x^3$. Write your function, derivative, initial guess, and each improved guess. Stop when you believe your solution is correct to the 9th decimal place.

$f(x) = 75 - x^3 - \sin 4x$

$f'(x) = -3x^2 - 4\cos 4x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

BECAUSE $\sin 4x$ IS BETWEEN -1 AND 1 AND $\sqrt[3]{75} \approx 4.2$,

I CHOOSE $x_0 = 4$.

$x_0 = 4$

$x_1 = 4.255559573$

$x_2 = 4.234927281$

$x_3 = 4.234761591$

$x_4 = 4.23476158$

x_5 SAME AS x_4

$$\boxed{\text{SOLUTION IS } x = 4.234761580}$$

3. (15 points) The function g and its first two derivatives are shown below.

$$g(x) = \frac{x^2+x+1}{x^2+1}$$

$$g'(x) = \frac{1-x^2}{(x^2+1)^2} \qquad g''(x) = \frac{2x^3-6x}{(x^2+1)^3}$$

(a) Determine all horizontal asymptotes of the graph of g.

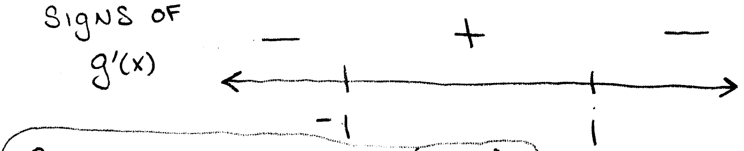
$$\lim_{x\to\pm\infty} \frac{x^2+x+1}{x^2+1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\pm\infty}\frac{1+\frac{1}{x}+\frac{1}{x^2}}{1+\frac{1}{x^2}} = \frac{1}{1}$$

$\boxed{\text{H. A. IS } y=1}$

(b) Find open intervals on which g is increasing/decreasing.

$g'(x)=0 \Rightarrow x=\pm 1$

$g'(x)$ DNE NEVER



g IS INCREASING ON $(-1,1)$ AND DECREASING ON $(-\infty,-1)\cup(1,\infty)$

(c) Find all relative extreme values of g.

$g(-1)=\frac{1}{2}$ IS A REL MIN

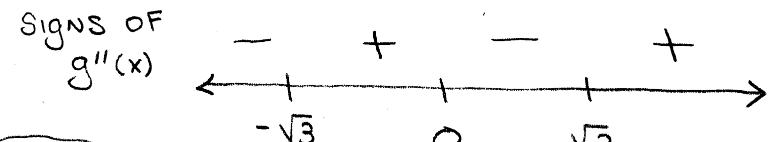
$g(1)=\frac{3}{2}$ IS A REL MAX

(d) Find open intervals on which the graph of g is concave up/down.

$g''(x)=0 \Rightarrow 2x^3-6x=0$

$2x(x^2-3)=0$

$x=0,\ x=\pm\sqrt{3}$



GRAPH IS CD ON $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$ AND CU ON $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$

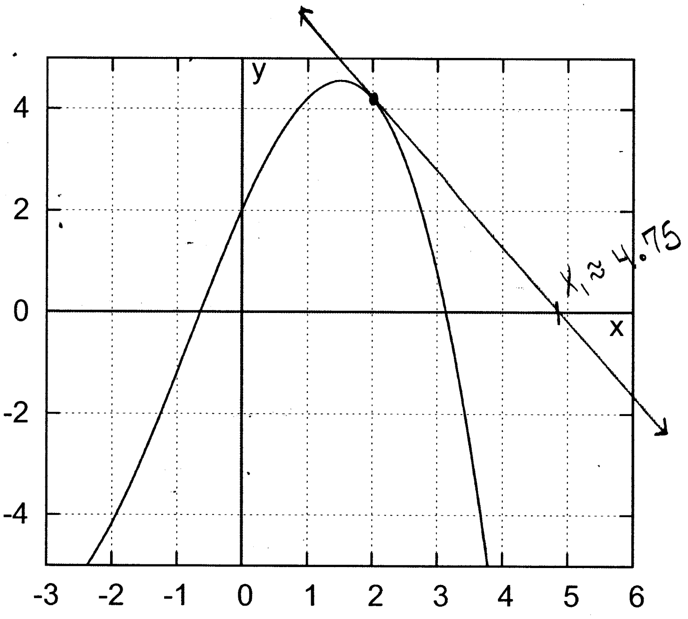
$g''(x)$ DNE NEVER

(e) Find all points of inflection of the graph of g.

$(-\sqrt{3}, 0.567)$, $(0,1)$, AND $(\sqrt{3}, 1.433)$ ARE INFLECTION PTS BECAUSE CONCAVITY CHANGES

4. (4 points) The graph of $y = f(x)$ is shown below. Suppose you use Newton's method, starting with $x_0 = 2$, to approximate a solution of $f(x) = 0$. Which one of the following numbers would be closest to x_1? Explain your reasoning.

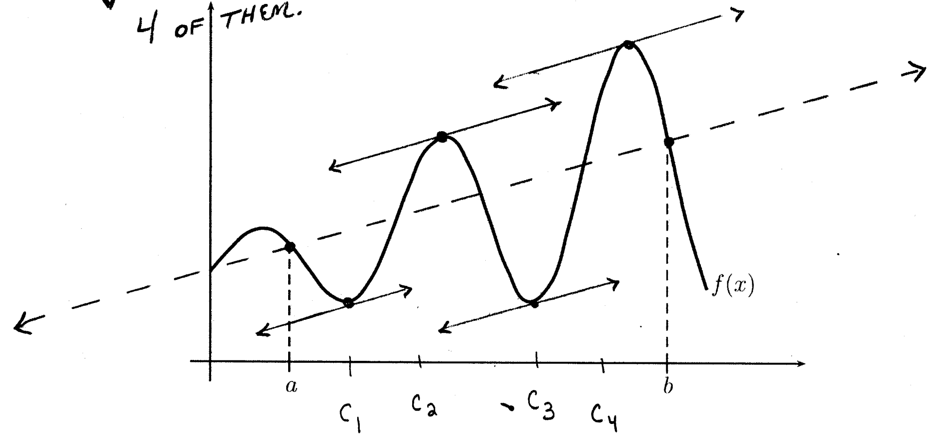
(a) 3.13
[✓ (b) 4.75]
(c) 3.93
(d) 0



LINEARIZATION OF f AT X=2 CROSSES X-AXIS AT ABOUT 4.75

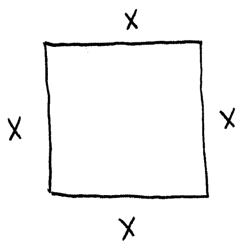
5. (4 points) Sketch the line tangent to the graph of f at **each** value c that is guaranteed by the Mean Value Theorem over the interval $[a, b]$.

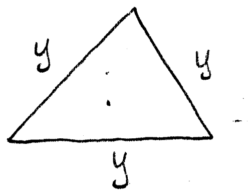
THERE ARE 4 OF THEM.



SECANT LINE THROUGH $(a, f(a))$ & $(b, f(b))$

6. (12 points) A wire of length 100 cm is cut into two pieces: one is bent to form a square and the other is bent to form an equilateral triangle. Where should the cut be made if the sum of the two areas is to be a minimum? (Hint: The area of an equilateral triangle with side length a units is $\frac{\sqrt{3}}{4}a^2$ square units.)





$$4x + 3y = 100$$

MINIMIZE $X^2 + \frac{\sqrt{3}}{4} y^2 = \text{AREA}$

SUBJECT TO $\underbrace{4x + 3y = 100}$

$$X = \frac{100-3y}{4}$$

$$\text{AREA} = A(y) = \left(\frac{100-3y}{4}\right)^2 + \frac{\sqrt{3}}{4} y^2 , \quad 0 \le y \le \frac{100}{3}$$

$$A'(y) = 2\left(\frac{100-3y}{4}\right)\left(-\frac{3}{4}\right) + \frac{\sqrt{3}}{2} y$$

$$= -\frac{300}{8} + \left(\frac{9}{8} + \frac{\sqrt{3}}{2}\right) y = -\frac{300}{8} + \frac{9+4\sqrt{3}}{8} y$$

$$A'(y) = 0 \Rightarrow (9+4\sqrt{3})y - 300 = 0$$

$$\Rightarrow y = \frac{300}{9+4\sqrt{3}} \approx 18.8345$$

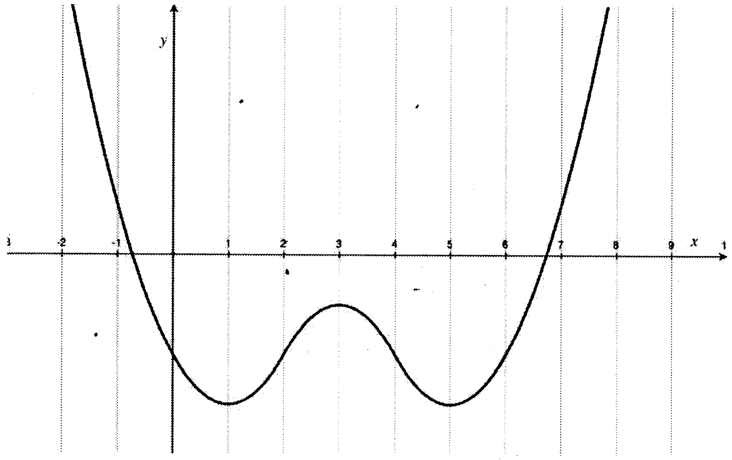
$A(0) = 625$

$A(18.8345) = 271.8528$ ← ABS MIN

$A(\frac{100}{3}) = 481.1252$

$$\Rightarrow \boxed{\begin{array}{l} y = 18.8345 \text{ IN} \\ X = 10.8741 \text{ IN} \\ \text{MAKE CUT AT } 43.4965 \text{ IN} \end{array}}$$

7. (12 points) The graph of g is shown below.



(a) Determine open intervals on which $g'(x)$ is positive.

$g'(x)$ pos $\Leftrightarrow$ $g(x)$ INCREASING $\Rightarrow$ $\boxed{(1,3) \cup (5,\infty)}$

(b) Determine open intervals on which $g'(x)$ is negative.

$g'(x)$ NEG $\Leftrightarrow$ $g(x)$ DECREASING $\Rightarrow$ $\boxed{(-\infty, 1) \cup (3,5)}$

(c) Find any x-values for which $g'(x) = 0$.

$\boxed{X = 1, \ X = 3, \ X = 5}$

(d) Determine open intervals on which $g''(x)$ is positive.

$g''(x)$ pos $\Leftrightarrow$ GRAPH IS CU $\Rightarrow$ $\boxed{(-\infty, 2) \cup (4,\infty)}$

(e) Determine open intervals on which $g''(x)$ is negative.

$g''(x)$ NEG $\Leftrightarrow$ GRAPH IS CD $\Rightarrow$ $\boxed{(2,4)}$

(f) Find any x-values for which $g''(x) = 0$.

INFLECTION PTS $\boxed{X = 2, \ X = 4}$

8. (6 points) Evaluate each indefinite integral.

(a) $\int (3x^2 - 7x + 1)\,dx$

$$= \boxed{x^3 - \frac{7}{2}x^2 + x + C}$$

(b) $\int (2\sqrt{t} - \sec^2 t)\,dt$ $= \int \left(2t^{1/2} - \sec^2 t\right) dt$

$$= \boxed{\frac{4}{3}t^{\frac{3}{2}} - \tan t + C}$$

9. (6 points) Use a linearization to estimate $\frac{1}{\sqrt[3]{8.1}}$.

$f(x) = x^{-1/3}$

$c = 8, \quad f(c) = f(8) = \frac{1}{2}$

$f'(x) = -\frac{1}{3}x^{-4/3}$

$f'(c) = f'(8) = -\frac{1}{3}(2)^{-4} = -\frac{1}{48}$

$L(x) = \frac{1}{2} - \frac{1}{48}(x-8)$

$\frac{1}{\sqrt[3]{8.1}} \approx L(8.1) = \frac{1}{2} - \frac{0.1}{48}$

$\approx \boxed{0.4979}$

Math 171 - Test 3b

November 13, 2014

Name key

Score ______

Show all work to receive full credit. Supply explanations where necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

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1. (4 points) Evaluate the limit: $\lim_{x\to-\infty} \frac{7x-13}{\sqrt{4x^2-3x-8}} \cdot \frac{-\frac{1}{x}}{\frac{1}{\sqrt{x^2}}}$

$\sqrt{x^2} = -x$

For $x < 0$.

$\therefore$

$\frac{1}{\sqrt{x^2}} = -\frac{1}{x}$

$$= \lim_{x\to-\infty} \frac{-7 + \frac{13}{x}\to 0}{\sqrt{4 - \frac{3}{x}\to 0 - \frac{8}{x^2}\to 0}}$$

$$= -\frac{7}{\sqrt{4}} = \boxed{-\frac{7}{2}}$$

2. (4 points) A right circular cylinder of height exactly 9 in has volume given by $V = 9\pi r^2$. Use differentials to estimate the propagated error if the radius is measured to be 5 in with an error of at most 0.04 in.

$dV = 18\pi r\, dr$

$\Downarrow$

$\Delta V \approx 18\pi r\, \Delta r$

$\Delta V \approx 18\pi(5)(0.04) \approx \boxed{11.31 \text{ in}^3}$

3. (6 points) Find the absolute extreme values of $f(x) = \frac{x^2+3x-2}{x-3}$ over the interval $[4, 10]$.

$$f'(x) = \frac{(x-3)(2x+3) - (x^2+3x-2)(1)}{(x-3)^2} = \frac{x^2-6x-7}{(x-3)^2}$$

$$= \frac{(x-7)(x+1)}{(x-3)^2} = 0 \Rightarrow \cancel{x=-1},\ \underline{\underline{x=7}}$$

EVAL f AT CRIT #'S

AND ENDPTS...

$$\boxed{\begin{array}{l} f(7) = 17 \leftarrow \text{ABS MIN} \\ f(4) = 26 \leftarrow \text{ABS MAX} \end{array}}$$

$$f(10) = 18.2857\ldots$$

4. (4 points) Find a function g such that $g'(x) = \frac{x^3-1}{x^2}$ and $g(1) = 8$.

$$g'(x) = x - x^{-2}$$

$$g(x) = \frac{1}{2}x^2 + x^{-1} + C$$

$$g(1) = \frac{1}{2} + 1 + C = 8$$

$$\Rightarrow C = 6.5$$

$$\boxed{g(x) = \frac{1}{2}x^2 + \frac{1}{x} + \frac{13}{2}}$$

5. (7 points) Consider the function $f(x) = (x-9)^3(x-1)$. Find open intervals on which the graph of f is concave up/down. Identify all points of inflection of the graph of f.

$$f'(x) = 3(x-9)^2(x-1) + (x-9)^3 = (x-9)^2(3x-3+x-9)$$

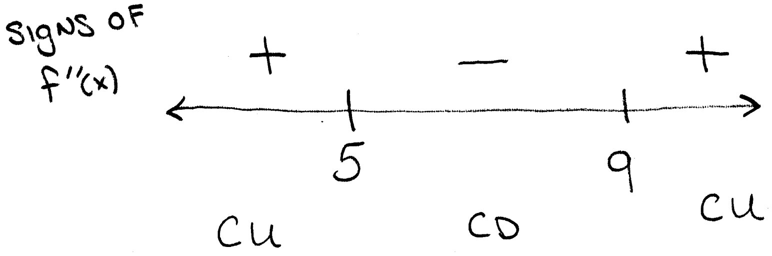
$$= (x-9)^2(4x-12)$$

$$f''(x) = 2(x-9)(4x-12) + (x-9)^2(4)$$

$$= (X-9)(8x-24+4x-36) = (x-9)(12x-60)$$

$$= (x-9)(12)(x-5) = 0$$

$$\Rightarrow \quad X = 9 \text{ OR } X = 5$$



GRAPH IS CONCAVE UP ON $(-\infty, 5) \cup (9, \infty)$

GRAPH IS CONCAVE DOWN ON $(5, 9)$

$(5, -256)$ & $(9, 0)$ ARE INFLECTION POINTS