Math 172 - Quiz 7

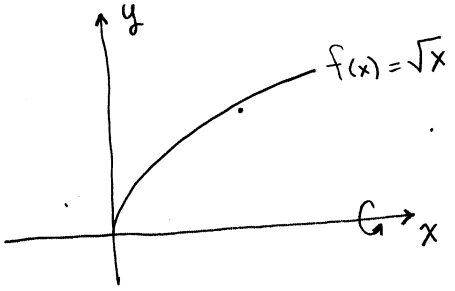
October 11, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Find the area of the surface obtained by revolving the graph of $f(x) = \sqrt{x}$ on the interval $[0, 2]$ about the x-axis. Evaluate your integral by hand.



$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$$\text{Area} = 2\pi \int_0^2 \sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx$$

$$= 2\pi \int_0^2 \sqrt{x+\frac{1}{4}}\,dx = 2\pi \int_{1/4}^{9/4} \sqrt{u}\,du$$

$u = x + \frac{1}{4}$

$du = dx$

$$= 2\pi \left(\frac{2}{3}u^{3/2}\right)\Big|_{1/4}^{9/4}$$

$$= \boxed{\frac{13\pi}{3}}$$

2. (5 points) A bucket weighing 4 lb when empty and attached to a rope of negligible weight is used to draw water from a well that is 30 ft deep. Initially, the bucket contains 40 lb of water, but as it is pulled up at a constant rate of 2 ft/sec, the water leaks out of the bucket at the rate of 0.2 lb/sec. Find the work done in pulling the bucket to the top of the well.

Bucket & water weighs 44 lb at start

Leaks at $0.2/2$ lb/ft.

Weight at height: $F = 44 - 0.1y$

$$\text{Work} = \int_0^{30} (44 - 0.1y)\,dy$$

$$= 44y - \frac{0.1}{2}y^2\Big|_0^{30}$$

$$= 30(44 - 1.5) = \boxed{1275 \text{ ft lb}}$$

Modeled as chain...

$(41 \text{ lb})(30 \text{ ft})$

$$+ \int_0^{30} 0.1\,dy\,(30-y) = 1275 \text{ ft lb}$$

(0.1 lb, $(30-y)$ ft)