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Show all work to receive full credit. Supply explanations where necessary. Unless otherwise stated, you may use your calculator to evaluate any definite integrals.

1. ( 5 points) Use the definitions of the hyperbolic functions (in terms of exponential functions) to prove the following identity:

$$
\sinh 2 x=2 \sinh x \cosh x
$$

2. (5 points) Using the definitions of the hyperbolic functions (in terms of exponential functions) and the quotient rule, show that $\frac{d}{d x}[\operatorname{sech} x]=-\operatorname{sech} x \tanh x$.
3. (5 points) Use the derivative formula, $\frac{d}{d x}\left[\tanh ^{-1} x\right]=\frac{1}{1-x^{2}}$, to determine the derivative of $y=\tanh ^{-1} \sqrt{x}$.
4. (15 points) Consider the region between the graphs of $y=x^{3}$ and $y=2 x-x^{2}$. The graphs are shown below.

(a) Find the $x$-coordinates of the points at which the graphs intersect.
(b) Find the area of the combined regions enclosed by the graphs. Evaluate your definite integral(s) by hand.
5. (10 points) Find the length of the graph of $f(x)=\ln x-\frac{x^{2}}{8}$ from $x=1$ to $x=2$. Evaluate your definite integral by hand. Then use your calculator to evaluate the integral to check your work.
6. (8 points) The 1 st quadrant region bounded by the graphs $y=x-x^{3}, x=1$, and $y=1$ is rotated about the $x$-axis to generate a solid of revolution. Find the volume of the solid. Use your calculator to evaluate the required integral(s).
7. (12 points) The region bounded by the graphs of $f(x)=\sqrt{x}$ and $y=x^{2}$ is rotated about the given line to generate a solid of revolution. Set up the definite integral that gives the volume of the solid. Then use your calculator to evaluate the integral.
(a) $x=1$
(b) $x=0$
8. (6 points) The expression $2 \pi \int_{0}^{6}(x+2) \sqrt{6-x} d x$ gives the volume of a solid of revolution. Identify the plane region being rotated and the axis of revolution.
9. (14 points) Water weighs 62.4 pounds per cubic foot. An open tank filled with water has the shape of a right circular cone with its point down. The tank is 8 feet across the top and 6 feet high. How much work is done in emptying the tank by pumping the water to a platform 2 feet above the top edge of the tank?
10. (20 points) A thin plate is lying in the 1st quadrant bounded by the graphs of $y=6-x^{2}$, $y=x$, and $x=0$. The density of the plate at the point $(x, y)$ is given by $\rho(x)=1+2 x$. Find the center of mass of the plate. You may use your calculator to evaluate the required integrals.
