

Math 172 - Test 3

November 22, 2017

Name key _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Evaluate the limit: $\lim_{x \rightarrow 3^+} \left(\frac{18}{x^2 - 9} - \frac{x}{x - 3} \right)^{\infty - \infty}$

$$\lim_{x \rightarrow 3^+} \left(\frac{18 - x(x+3)}{x^2 - 9} \right)$$

$$= \lim_{x \rightarrow 3^+} \frac{-x^2 - 3x + 18}{x^2 - 9} \quad \% \quad \left. \begin{array}{l} \text{Or use} \\ \text{L'Hopital's rule} \end{array} \right\}$$

$$= \lim_{x \rightarrow 3^+} \frac{-(x-3)(x+6)}{(x-3)(x+3)} = -\frac{9}{6} = \boxed{-\frac{3}{2}}$$

2. (8 points) Explain why the integral $\int_0^\infty \frac{3}{16+x^2} dx$ is improper. Write it as a limit of proper integrals and evaluate.

↑ THE INTEGRATION INTERVAL IS UNBOUNDED.

$$\lim_{c \rightarrow \infty} \int_0^c \frac{3}{16+x^2} dx = \lim_{c \rightarrow \infty} \left(\frac{3}{4} \tan^{-1} \frac{x}{4} \right) \Big|_0^c$$

$$= \lim_{c \rightarrow \infty} \frac{3}{4} \tan^{-1} \frac{c}{4}$$

$$= \frac{3}{4} \cdot \frac{\pi}{2} = \boxed{\frac{3\pi}{8}}$$

3. (8 points) Integrate: $\int 5 \sec^6 8y \tan 8y dy$

$$u = 8y \quad \frac{5}{8} \int \sec^5 u \sec u \tan u du$$

$$du = 8dy$$

$$\omega = \sec u$$

$$d\omega = \sec u \tan u du$$

$$\frac{5}{8} \int \omega^5 d\omega = \frac{5}{48} \omega^6 + C$$

$$= \boxed{\frac{5}{48} \sec^6 8y + C}$$

4. (10 points) Integrate: $\int_0^{1/2} \cos^{-1} x dx$

$$u = \cos^{-1} x \quad du = \frac{-1}{\sqrt{1-x^2}} dx$$

$$dv = dx \quad v = x$$

$$\int_0^{1/2} \cos^{-1} x dx = x \cos^{-1} x \Big|_0^{1/2} + \int_0^{1/2} \frac{x}{\sqrt{1-x^2}} dx$$

$$\omega = 1-x^2$$

$$d\omega = -2x dx$$

$$-\frac{1}{2} d\omega = x dx$$

$$-\frac{1}{2} \int_1^{3/4} \omega^{-1/2} d\omega$$

$$= -\omega^{1/2} \Big|_1^{3/4}$$

$$= x \cos^{-1} x \Big|_0^{1/2} - \sqrt{\omega} \Big|_1^{3/4}$$

$$= \left[\frac{1}{2} \left(\frac{\pi}{3} \right) - 0 \right] - \left[\sqrt{\frac{3}{4}} - 1 \right] = \boxed{\frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1}$$

$$5. \text{ (12 points) Integrate: } \int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8} dx = \int 2x + \frac{x+5}{(x-4)(x+2)} dx$$

$$\begin{array}{c} \frac{\partial x}{x^2 - 2x - 8} \\ \frac{\partial x^3 - 4x^2 - 15x + 5}{\partial x^3 - 4x^2 - 16x} \\ \hline x+5 \end{array}$$

$$\frac{x+5}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$$x+5 = A(x+2) + B(x-4)$$

$$\begin{aligned} x=-2: \quad 3 &= -6B \\ x=4: \quad 9 &= 6A \end{aligned} \quad \left. \begin{array}{l} B = -\frac{1}{2} \\ A = \frac{3}{2} \end{array} \right\}$$

$$\begin{aligned} \int \frac{x+5}{(x-4)(x+2)} dx &= \int \left(\frac{3/2}{x-4} - \frac{1/2}{x+2} \right) dx \end{aligned}$$

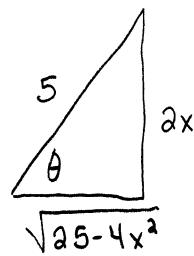
$$\int 2x + \frac{3/2}{x-4} - \frac{1/2}{x+2} dx$$

$$= x^2 + \frac{3}{2} \ln|x-4| - \frac{1}{2} \ln|x+2| + C$$

6. (10 points) Integrate: $\int \sqrt{25 - 4x^2} dx$

$$2x = 5 \sin \theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$2dx = 5 \cos \theta d\theta$$



$$\frac{5}{2} \int \sqrt{25 - 25 \sin^2 \theta} \cos \theta d\theta$$

$$= \frac{25}{2} \int |\cos \theta| \cos \theta d\theta = \frac{25}{2} \int \cos^2 \theta d\theta$$

$$= \frac{25}{4} \int (1 + \cos 2\theta) d\theta = \frac{25}{4} \left(\theta + \frac{1}{2} \sin 2\theta \right) + C$$

$$= \frac{25}{4} (\theta + \sin \theta \cos \theta) + C = \boxed{\frac{25}{4} \left(\sin^{-1} \frac{2x}{5} + \frac{2x}{5} \cdot \frac{\sqrt{25 - 4x^2}}{5} \right) + C}$$

7. (8 points) Evaluate the limit: $\lim_{x \rightarrow -\infty} x^2 e^x$ $\infty \cdot 0$

$$\left. \begin{array}{c} \lim_{x \rightarrow -\infty} \frac{x^2}{e^{-x}} \\ \infty/\infty \end{array} \right. \quad \left. \begin{array}{c} -\infty/-\infty \end{array} \right.$$

$$\left. \begin{array}{c} \text{L'Hôpital} \\ \{ \end{array} \right. = \lim_{x \rightarrow -\infty} \frac{2x}{-e^{-x}}$$

$$\left. \begin{array}{c} \text{L'Hôpital} \\ \{ \end{array} \right. = \lim_{x \rightarrow -\infty} \frac{2}{e^{-x}} = \boxed{0}$$

8. (8 points) Use a product-to-sum formula to evaluate the following integral.

$$\int \cos(3x) \cos(7x) dx$$

$$\frac{1}{2} \int (\cos 4x + \cos 10x) dx$$

$$= \frac{1}{2} \left[\frac{1}{4} \sin 4x + \frac{1}{10} \sin 10x \right] + C$$

$$\frac{1}{8} \sin 4x + \frac{1}{20} \sin 10x + C$$

9. (8 points) Integrate: $\int x^3 \sin 2x dx$

SIGNS	$u \& \text{ DERIVS}$	$\frac{dv}{dx} \text{ AND ANTS}$
+	x^3	$\sin 2x$
-	$3x^2$	$-\frac{1}{2} \cos 2x$
+	$6x$	$-\frac{1}{4} \sin 2x$
-	6	$\frac{1}{8} \cos 2x$
+	0	$\frac{1}{16} \sin 2x$

$$\begin{aligned}
 & -\frac{1}{2} x^3 \cos 2x + \frac{3}{4} x^2 \sin 2x \\
 & + \frac{6}{8} x \cos 2x - \frac{6}{16} \sin 2x \\
 & + C
 \end{aligned}$$

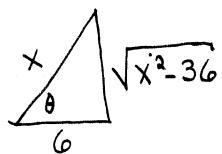
10. (3 points) Explain why the integral is improper: $\int_{-1}^4 \frac{1}{x^3} dx$

THE INTEGRAND HAS AN
INFINITE DISCONTINUITY (VERT. ASYMP.)
AT $x = 0$, WHICH IS INSIDE THE
INTEGRATION INTERVAL.

11. (4 points) Write the form of the partial fraction decomposition of $\frac{x}{x^3(x^2+9)^2(2x+1)}$.
Do not solve for the undetermined coefficients.

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+9} + \frac{Fx+G}{(x^2+9)^2} + \frac{H}{2x+1}$$

12. (5 points) After making the trigonometric substitution $x = 6 \sec \theta$, you evaluated an integral and obtained $\theta + \cot \theta + C$. Resubstitute and write your result in terms of the variable x .



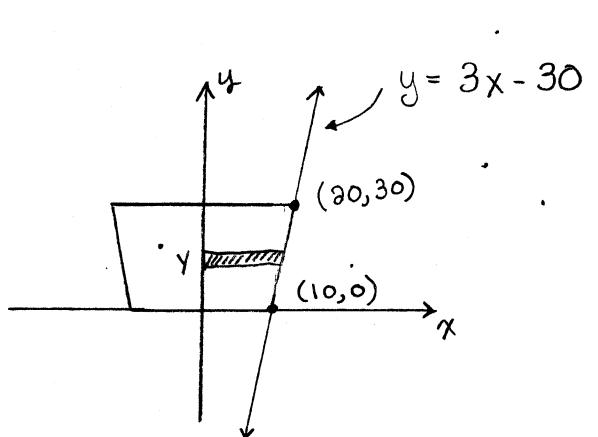
$$\sec \theta = \frac{x}{6}$$

$$\cos \theta = \frac{6}{x}$$

$$\theta + \cot \theta + C = \boxed{\sec^{-1} \frac{x}{6} + \frac{6}{\sqrt{x^2 - 36}} + C}$$

13. (8 points) (Take-home problem)

A large vertical dam in the shape of an isosceles trapezoid has a height of 30 m, a width of 20 m at its base, and a width of 40 m at the top. What is the total fluid force on the face of the dam when the reservoir is full? (Assume the water weighs 9807 N/m^3 .)



$$x = \frac{y+30}{3}$$

$$\int_0^{30} 9807(a)\left(\frac{y+30}{3}\right)(30-y) dy$$

$$= 6538 \int_0^{30} (900-y^2) dy$$

$$= 6538 \left(900y - \frac{1}{3}y^3 \right) \Big|_0^{30}$$

$$= 6538 \left(\frac{2}{3} \right) (27000)$$

$$= 117,684,000 \text{ N}$$