Math 172 - Test 3 November 22, 2017

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Evaluate the limit:
$$\lim_{x \to 3^+} \left(\frac{18}{x^2 - 9} - \frac{x}{x - 3} \right)$$

2. (8 points) Explain why the integral $\int_0^\infty \frac{3}{16+x^2} dx$ is improper. Write it as a limit of proper integrals and evaluate.

3. (8 points) Integrate: $\int 5 \sec^6 8y \tan 8y \, dy$

4. (10 points) Integrate: $\int_0^{1/2} \cos^{-1} x \, dx$.

5. (12 points) Integrate:

$$\int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8} \, dx.$$

6. (10 points) Integrate: $\int \sqrt{25 - 4x^2} \, dx.$

7. (8 points) Evaluate the limit: $\lim_{x \to -\infty} x^2 e^x$

8. (8 points) Use a product-to-sum formula to evaluate the following integral.

$$\int \cos(3x)\,\cos(7x)\,dx$$

9. (8 points) Integrate: $\int x^3 \sin 2x \, dx$

10. (3 points) Explain why the integral is improper: $\int_{-1}^{4} \frac{1}{x^3} dx$

11. (4 points) Write the form of the partial fraction decomposition of $\frac{x}{x^3(x^2+9)^2(2x+1)}$. Do not solve for the undetermined coefficients.

12. (5 points) After making the trigonometric substitution $x = 6 \sec \theta$, you evaluated an integral and obtained $\theta + \cot \theta + C$. Resubstitute and write your result in terms of the variable x.

13. (8 points) (Take-home problem)

A large vertical dam in the shape of an isosceles trapezoid has a height of $30 \,\mathrm{m}$, a width of $20 \,\mathrm{m}$ at its base, and a width of $40 \,\mathrm{m}$ at the top. What is the total fluid force on the face of the dam when the reservoir is full? (Assume the water weighs $9807 \,\mathrm{N/m^3}$.)