Math 172 - Final Exam A December 11, 2017

Name _____ Score _____

Show all work to receive full credit. Supply explanations where necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

 $\lim_{x \to \infty} x \left(e^{\frac{1}{2x}} - e^{\frac{1}{4x}} \right)$ 1. (5 points) Evaluate:

2. (5 points) Evaluate: $\int_{0}^{5} \frac{1}{25 - x^2} dx$

3. (a) (3 points) Determine the second Maclaurin polynomial for $f(x) = e^x$.

(b) (2 points) Now that you have a Maclaurin polynomial for $f(x) = e^x$, you can approximate $e^{\frac{1}{kx}}$ by substituting $\frac{1}{kx}$ in place of the x in your Maclaurin polynomial. Re-evaluate the limit in Problem #1 by using this idea.

4. (6 points) Determine the third Maclaurin polynomial for $g(x) = \tan x$. Then use it to approximate $\tan(\pi/30)$.

5. (3 points) The *n*th term of a sequence is given by $a_n = \frac{\ln(n^3)}{6n}$. Write the first four terms of the sequence. Then determine if the sequence converges or diverges.

- 6. (8 points) Consider the series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$.
 - (a) Use the logarithm laws to rewrite the terms of the series.

(b) Find a formula for the nth partial sum of the series.

(c) Does the series converge or diverge? Explain how you know.

(d) Explain why the *n*th term test does not apply to this series.

7. (18 points) Determine whether each series converges conditionally, converges absolutely, or diverges.

(a)
$$\sum_{n=1}^{\infty} \frac{4n}{2n^2 + 1}$$

(b)
$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$
 (Hint: Use ratio test.)

(c)
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n+1)}$$

Math 172 - Final Exam B

Name _____

December 11, 2017

Score _____

Show all work to receive full credit. Supply explanations where necessary. All integrals must be evaluated by hand, with work shown, unless otherwise stated.

1. (10 points) For $x \ge 1$, let $f(x) = \frac{100x\sqrt{x-1}}{(2x+1)^2}$. Determine f'(x) and then find an equation of the line tangent to the graph of f at x = 2. (Hint: Use logarithmic differentiation.)

- 2. (8 points) Let $f(x) = x^5 + 7x + 2$.
 - (a) Briefly explain how we can be certain that f has an inverse.
 - (b) Briefly explain why it would be difficult to find a formula for $f^{-1}(x)$.
 - (c) Compute $(f^{-1})'(10)$.

3. (6 points) Evaluate the indefinite integral:

 $\int x \sec x \tan x \, dx$

4. (12 points) Use a partial fraction decomposition to evaluate the following indefinite integral.

$$\int \frac{5x^2 + 3x + 22}{(x^2 + 9)(x - 1)} \, dx$$

- 5. Consider the 1st-quadrant region bounded by the graphs of $y = e^x$, y = 1, and x = 2.
 - (a) (8 points) The region is rotated about the x-axis to generate a solid of revolution. Find the volume of the solid. Evaluate the definite integral(s) by hand.

(b) (5 points) The region is rotated about the line x = 3 to generate a solid of revolution. Find the volume of the solid. Use your calculator to evaluate the definite integral(s).

6. (10 points) A thin plate covers the region between the graphs of $y = 2x^2 - 5x$ and y = x. The density of the plate at the point (x, y) is given by $\delta(x) = x$. Set up the integral that gives the plate's moment about the x-axis. Use your calculator to evaluate the integral.

7. (5 points) Evaluate the limit: $\lim_{x\to 0^+} x \ln x$

8. (6 points) Write the following improper integral as a limit of proper integrals and evaluate. (Hint: You'll need your result from above.)

 $\int_0^1 \ln x \, dx$

9. (10 points) Use an appropriate trigonometric substitution to evaluate the following definite integral.

$$\int_0^1 \frac{1}{(x^2+1)^{3/2}} \, dx$$

10. (5 points) Consider the series
$$\sum_{n=1}^{\infty} \frac{8}{3^n}$$
.

(a) Find the fourth partial sum of the series.

(b) Does the series converge or diverge? Explain how you know.

(c) If the series converges, find its sum.

11. (15 points) Determine whether each <u>series</u> converges or diverges. Show all work and explain your reasoning.

(a)
$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

(b)
$$\sum_{n=1}^{\infty} \ln \frac{1}{n}$$

(c)
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 - n}$$