MuPAD Light Version 2.5.3 — Examples

MuPAD is a computer algebra system originally developed in the early 1990's at the University of Paderborn. It is now developed in cooperation with SciFace Software.

1. (Comments) To type a comment, start a line with two slashes.

   - `// It's a great day to do math!`

2. (Numeric Computation) Find the exact and approximate values of $\cos(\pi/5)$.

   - `num:=cos(PI/5)`

     $$\frac{5^{1/2}}{4} + 1/4$$

   - `float(num)`

     $$0.8090169944$$

   - `DIGITS:=25`

     $$25$$

   - `float(num)`

     $$0.8090169943749474241022934$$

   - `delete(DIGITS)`

3. (Functions) Define f as a function and evaluate at the given point.

   $$f(x) = 2\sin^2 x \cos x, \quad x = \pi$$

   - `f:=x->2*sin(x)^2*cos(x)`

     $$x \rightarrow 2 * \sin(x)\hat{}2 * \cos(x)$$

   - `f(PI)`

     $$0$$

4. (Expressions) Define f as an expression and evaluate at the given point.

   $$f(x) = 2\sin^2 x \cos x, \quad x = \pi$$

   - `f:=2*sin(x)^2*cos(x)`

     $$2\ \cos(x)\ \sin(x)^2$$

   - `subs(f,x=PI)`

     $$2\ \cos(\mathrm{PI})\ \sin(\mathrm{PI})^2$$

   - `simplify(%)`

     $$0$$

5. (Graphing) Sketch the graphs of $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq 2\pi$.

   - `plotfunc2d(sin(x),cos(x),x=0..2*PI)`

     output omitted

6. (Inequalities) Find the solution set for the inequality $x^2 - 3x \geq 4$.

- `solve(x^2-3*x>=4,x)`

$$]-\text{infinity}, -1] \text{ union } [4, \text{infinity}[$$

7. (Linear Systems) Solve the following system of equations.

$$\begin{array}{rcrcrcr} 2x & + & 3y & - & z & = & 9 \\ -4x & + & y & + & 3z & = & 0 \\ 5x & - & 7y & - & 2z & = & -5 \end{array}$$

- `solve({2*x+3*y-z=9,-4*x+y+3*z=0,5*x-7*y-2*z=-5},{x,y,z})`

$$\{[x = 121/36,\ y = 73/36,\ z = 137/36]\}$$

8. (Limits) Compute the left- and right-hand limits at $x = 2$ for the function

$$f(x) = \begin{cases} x^2, & x < 2 \\ 3, & x = 2 \\ 4 - x, & x > 2 \end{cases}.$$

- `f:=piecewise([x<2,x^2],[x=2,3],[x>2,4-x])`

$$\text{piecewise}(x^2 \text{ if } x < 2,\ 3 \text{ if } x = 2,\ -x + 4 \text{ if } 2 < x)$$

- `limit(f,x=2,Left)`

$$4$$

- `limit(f,x=2,Right)`

$$2$$

- `limit(f,x=2)`

$$\text{undefined}$$

9. (Continuity) Find all points at which the following rational function is discontinuous.

$$R(x) = \frac{x - 2}{x^2 - 5x - 6}$$

- `discont((x-2)/(x^2-5*x-6),x)`

$$\{2, 3\}$$

10. (Limit at Infinity) Evaluate: $\lim_{x\to\infty} \frac{\sin x}{x}$.

- `limit(sin(x)/x,x=infinity)`

$$0$$

11. (Derivative by Definition) Use the limit definition of the derivative to find $g'(x)$ if $g(x) = \sqrt{2x+1}$.

- `g:=x->sqrt(2*x+1)`

$$x \to \text{sqrt}(2x+1)$$

- `limit((g(x+h)-g(x))/h,h=0)`

$$\frac{1}{(2x+1)^{1/2}}$$

12. (Derivative of an Expression) Find the slope of the line tangent to the graph of $y = x + \frac{2}{x}$ at the point $(1, 3)$.

- `y:=x+2/x`

$$x + \frac{2}{x}$$

- `dydx:=diff(y,x)`

$$1 - \frac{2}{x^2}$$

- `subs(dydx,x=1)`

$$-1$$

13. (2nd Derivative of a Function) Find the 2nd derivative of $f(x) = x \sin x$.

- `f:=x->x*sin(x)`

$$x \to x * \sin(x)$$

- `f''(x)`

$$2\cos(x) - x\sin(x)$$

14. (Graphing) Sketch the graph of $y = \sin(x)/x$ and label the x-axis in units of π.

- `plotfunc2d(Ticks=[Steps=[PI,3],Automatic],sin(x)/x,x=0..20)`

output omitted

15. (Linearization) Find the linearization, $L(x)$, of the function $f(x) = \sqrt{x^2+9}$ at $x = -4$. Sketch the graph of both f and L near $x = -4$.

- `f:=sqrt(x^2+9); df:=diff(f,x)`

$$(x^2+9)^{1/2}$$

$$\frac{x}{(x^2+9)^{1/2}}$$

- `x0:=-4; fx0:=subs(f,x=x0); dfx0:=subs(df,x=x0)`

$$-4$$

$$5$$

$$-4/5$$

- `L:=fx0+dfx0*(x-x0)`

$$9/5 - \frac{4x}{5}$$

- `plotfunc2d(f,L,x=-8..0)`

output omitted

16. (Newton's Method) Starting with $x_0 = 1$, take five steps of Newton's method to approximate a solution of $x - \cos x = 0$.

- `f:=x->x-cos(x)`

$$x \to x - \cos(x)$$

- `x:=1.0`

$$1.0$$

- `for i from 1 to 5 do x:=x-f(x)/f'(x); print(x) end_for:`

$$0.7503638678$$

$$0.7391128909$$

$$0.7390851334$$

$$0.7390851332$$

$$0.7390851332$$

17. (Regression) Fit the following data to a model of the form $y = A + B\ln x + C/x^2$.

$$(1,1),(4,2),(11,3),(31,4),(83,5),(227,6)$$

- `stats::reg([1,4,11,31,83,227],[1,2,3,4,5,6],A+B*ln(x)+C/x^2,[x],[A,B,C])`

$$[[0.5971200686, 0.995516558, 0.4028178245], 0.0004391734841]$$

18. (Riemann Sums) Partition the interval $[1, 5]$ into 20 subintervals of equal width and use right endpoints to find the corresponding Riemann sum for the function $f(x) = 1/x$.

- `export(student)`
- `f:=1/x`

$$\frac{1}{x}$$

- `riemann(f,x=1..5,20,Right)`

$$\frac{\text{sum}\left(\frac{1}{\frac{i1}{5}+1},\ i1 = 1..20\right)}{5}$$

- `float(%)`

$$1.532624844$$

- `plotRiemann(f,x=1..5,20,Right)`

$$\text{plot::Group}()$$

- `plot(%)`

output omitted

19. (Indefinite Integrals) Evaluate the indefinite integral $\int \frac{2z\,dz}{\sqrt[3]{z^2+1}}$.

- `int(2*z/(z^2+1)^(1/3),z)`

$$\frac{\frac{3z^2}{2} + 3/2}{(z^2+1)^{1/3}}$$

20. (Partial Fraction Decomposition) Find the partial fraction decomposition of

$$R(x) = \frac{1}{x^2 - x}.$$

- `partfrac(1/(x^2-x))`

$$\frac{1}{x-1} - \frac{1}{x}$$

- `normal(%)`

$$\frac{1}{x^2 - x}$$

21. (Finite Sums) Find the 100th harmonic number.

- `sum(1/n,n=1..100)`

output omitted

- `float(%)`

$$5.187377518$$

22. (Vector Operations) Let $\vec{u} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{v} = -3\hat{\imath} + 7\hat{k}$. Find $\vec{u} \cdot \vec{v}$, $\vec{u} \times \vec{v}$, and the angle between $\vec{u}$ and $\vec{v}$.

- `export(linalg)`
- `u:=matrix(3,1,[3,2,-1]); v:=matrix(3,1,[-3,0,7])`

$$\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}$$

- `scalarProduct(u,v)`

$$-16$$

- `crossProduct(u,v)`

$$\begin{bmatrix} 14 \\ -18 \\ 6 \end{bmatrix}$$

- `angle(u,v)`

$$\mathrm{PI} \; - \arccos\left(\frac{4 \;\; 14^{1/2}\, 58^{1/2}}{203}\right)$$

23. (Partial Derivative) Find $f_x(1, \pi, 2)$ if $f(x, y, z) = y \sin(xyz)$.

- `diff(y*sin(x*y*z),x)`

$$y^2 z \cos(xyz)$$

- `subs(%,x=1,y=PI,z=2)`

$$2\,\mathrm{PI}^2 \;\cos(2\,\mathrm{PI})$$

- `simplify(%)`

$$2\,\mathrm{PI}^2$$

- `float(%)`

$$19.7392088$$

24. (Higher-Order Partial Derivative) Compute f_{xxyz} if $f(x, y, z) = \sin(3x + yz)$.

- `f:=sin(3*x+y*z)`

$$\sin(3x + yz)$$

- `diff(f,x,x,y,z)`

$$9yz \sin(3x + yz) - 9\cos(3x + yz)$$

25. (Implicit Differentiation) Given the equation $xy + z^3x - 2yz = 0$, find $\partial z/\partial x$ at the point $(1, 1, 1)$.

- `f:=x*y+z^3*x-2*y*z`

$$xy - 2yz + xz^3$$

- `dzdx:=-diff(f,x)/diff(f,z)`

$$-\frac{y + z^3}{3xz^2 - 2y}$$

- `subs(dzdx,x=1,y=1,z=1)`

$$-2$$

26. (Critical Points) Let $V(y, z) = 108yz - 2y^2z - 2yz^2$. Find all points for which $V_y(y, z)$ and $V_z(y, z)$ are simultaneously zero.

- `V:=108*y*z-2*y^2*z-2*y*z^2`

$$108yz - 2yz^2 - 2y^2z$$

- `solve({diff(V,y)=0,diff(V,z)=0},{y,z})`

$$\{[y = 0, z = 0], [y = 18, z = 18], [y = 0, z = 54], [y = 54, z = 0]\}$$

27. (Gradient Vector) Compute ∇f at the given point.

$$f(x, y, z) = x^2 + y^2 - 2z^2 + z \ln x, \quad (1, 1, 1)$$

- `export(linalg)`
- `f:=x^2+y^2-2*z^2+z*ln(x)`

$$z \ln(x) + x^2 + y^2 - 2z^2$$

- `gradf:=grad(f,[x,y,z])`

$$\begin{bmatrix} 2x + \frac{z}{x} \\ 2y \\ -4z + \ln(x) \end{bmatrix}$$

- `subs(gradf,x=1,y=1,z=1)`

$$\begin{bmatrix} 3 \\ 2 \\ \ln(1) - 4 \end{bmatrix}$$

- `simplify(%)`

$$\begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix}$$

28. (Curvature) Find the curvature of $\vec{r}(t) = (2t - \sin t)\hat{\imath} + (2 - 2\cos t)\hat{\jmath}$ at the point where $t = 3\pi/2$.

- `export(linalg)`
- `assume(t,Type::Real)`

Type::Real

- `r:=matrix[2,1,[2*t-sin(t),2-2*cos(t)])`

$$\begin{bmatrix} 2t - \sin(t) \\ -2\cos(t) + 2 \end{bmatrix}$$

- `v:=diff(r,t)`

$$\begin{bmatrix} -\cos(t) + 2 \\ 2\sin(t) \end{bmatrix}$$

- `mag_v:=norm(v,2)`

$$(\cos(t)^2 - 4\cos(t) + 4\sin(t)^2 + 4)^{1/2}$$

- `T:=v/mag_v`

output omitted

- `K:=norm(diff(T,t),2)/mag_v`

output omitted

- `curvature:=subs(K,t=3*PI/2)`

output omitted

- `simplify(curvature)`

$$\frac{2^{1/2}}{16}$$

29. (Surface of Revolution) Sketch the graph of the surface obtained by rotating the graph of $y = \sqrt{x}$, $0 \leq x \leq 1$, about the x-axis.

- `f:=sqrt(x)`

$$x^{1/2}$$

- `plot(plot::xrotate(f,x=0..1))`

output omitted

30. (Directional Derivatives) Find the directional derivative of

$$f(x, y) = 2xy - 3y^2$$

at $(5, 5)$ in the direction of $\vec{u} = 4\hat{\imath} + 3\hat{\jmath}$.

- `export(linalg)`
- `f:=2*x*y-3*y^2`

$$2xy - 3y^2$$

- `u:=matrix(2,1,[4,3])`

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

- `gradf:=subs(grad(f,[x,y]),x=5,y=5)`

$$\begin{bmatrix} 10 \\ -20 \end{bmatrix}$$

- `scalarProduct(gradf,u)/norm(u,2)`

$$-4$$

31. (Ordinary Differential Equations) Solve the following differential equation:

$$xy' + 2y = xe^{x/2}$$

- `eq:=ode(x*y'(x)+2*y(x)=x*exp(x/2),y(x))`

$$\text{ode}\left(2\,y(x) + x\ \text{diff}(y(x), x) - x\exp\left(\frac{x}{2}\right), y(x)\right)$$

- `solve(eq)`

$$\left(\frac{c1}{x^2} + 2\exp(x)^{1/2} - \frac{8\exp(x)^{1/2}}{x} + \frac{16\exp(x)^{1/2}}{x^2}\right)$$

32. (Initial Value Problems) Solve the following initial value problem:

$$y'' + 2y' + y = e^{2x}; \qquad y(0) = 2,\ y'(0) = 1$$

- `eq:=ode({y''(x)+2*y'(x)+y(x)=exp(2*x),y(0)=2,y'(0)=1},y(x))`

$$\text{ode}(\{y(0) = 2, D(y)(0) = 1, y(x) + \cdots - \exp(2x)\}, y(x))$$

- `solve(eq)`

$$\left(\frac{17\exp(-x)}{9} + \frac{\exp(x)^2}{9} + \frac{8x\exp(-x)}{3}\right)$$

33. (Laplace Transforms) Find the Laplace transform of $f(t) = t^2 - 1 + \cos t$.

- `export(transform)`
- `laplace(t^2-1+cos(t),t,s)`

$$\frac{2}{s^3} - \frac{1}{s} + \frac{s}{s^2+1}$$

- `invlaplace(%,s,t)`

$$\cos(t) + t^2 - 1$$

34. (One Variable Statistics) Find the mean, median, mode, and standard deviation of the data set given below.

$$\{92.5, 43, 78, 82, 57.5, 63, 78, 91, 84.5, 68\}$$

- `export(stats)`
- `data:=[92.5,43,78,82,57.5,63,78,91,84.5,68]`

$$[92.5, 43, 78, 82, 57.5, 63, 78, 91, 84.5, 68]$$

- `mean(data)`

$$73.75$$

- `median(data)`

$$78$$

- `modal(data)`

$$[78], 2$$

- `stdev(data)`

$$15.73080841$$

- `stdev(data,Population)`

$$14.92355521$$

35. (Surface Plots) Sketch the surface defined by $z = \sin xy$.

- `plotfunc3d(sin(x*y),x=-PI..PI,y=-PI..PI,Grid=[200,200])`

output omitted

36. (Parametric Plots) Sketch the space curve defined by the parametric equations.

$$x = \sin t, \quad y = \cos t, \quad z = t$$

- `p1:=plot::Curve3d([sin(t),cos(t),t],t=0..4*PI)`

plot::Curve3d([sin(t), cos(t), t], t = 0..4 PI)

- `plot(p1)`

output omitted

37. (Matrix Operations) Given the two matrices A and B as shown below, find $A + 2B$, A^T, A^TA, and B^{-1}.

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 2 & -2 \\ 0 & -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & 3 \\ -1 & -3 & -5 \end{bmatrix}$$

- `A:=matrix(3,3,[[1,0,4],[3,2,-2],[0,-1,2]])`

$$\begin{bmatrix} 1 & 0 & 4 \\ 3 & 2 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

- `B:=matrix(3,3,[[1,1,1],[3,0,3],[-1,-3,-5]])`

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & 3 \\ -1 & -3 & -5 \end{bmatrix}$$

- `A+2*B`

$$\begin{bmatrix} 3 & 2 & 6 \\ 9 & 2 & 4 \\ -2 & -7 & -8 \end{bmatrix}$$

- `C:=linalg::transpose(A)`

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & -1 \\ 4 & -2 & 2 \end{bmatrix}$$

- `C*A`

$$\begin{bmatrix} 10 & 6 & -2 \\ 6 & 5 & -6 \\ -2 & -6 & 24 \end{bmatrix}$$

- `B^(-1)`

$$\begin{bmatrix} 3/4 & 1/6 & 1/4 \\ 1 & -1/3 & 0 \\ -3/4 & 1/6 & -1/4 \end{bmatrix}$$

38. (Gauss-Jordan Elimination) Use Gauss-Jordan elimination to solve the following system of equations.

$$\begin{array}{rcrcrcr} 2x & + & 3y & - & z & = & 9 \\ -4x & + & y & + & 3z & = & 0 \\ 5x & - & 7y & - & 2z & = & -5 \end{array}$$

- `M:=matrix(3,4,[[2,3,-1,9],[-4,1,3,0],[5,-7,-2,-5]])`

$$\begin{bmatrix} 2 & 3 & -1 & 9 \\ -4 & 1 & 3 & 0 \\ 5 & -7 & -2 & -5 \end{bmatrix}$$

- `linalg::gaussJordan(M)`

$$\begin{bmatrix} 1 & 0 & 0 & 121/36 \\ 0 & 1 & 0 & 73/36 \\ 0 & 0 & 1 & 137/36 \end{bmatrix}$$

39. (Nonlinear Equations) Sketch the graphs of the following equations and numerically approximate a point of intersection of the graphs.

$$4x^2 + y^2 - 4 = 0$$

$$x + y - \sin(x - y) = 0$$

- `f:=4*x^2+y^2-4; g:=x+y-sin(x-y)`

$$4x^2 + y^2 - 4$$

$$x + y - \sin(x - y)$$

- `p1:=plot::implicit(f=0,x=-2..2,y=-2..2)`

plot::Group()

- `p2:=plot::implicit(g=0,x=-2..2,y=-2..2)`

plot::Group()

- `plot(p1,p2)`

output omitted

- `numeric::solve({f=0,g=0},{x=1,y=0})`

$$\{[y = -0.1055304923, x = 0.9986069441]\}$$

40. (ODE Direction Fields) Sketch the direction field associated with the differential equation

$$\frac{dy}{dx} = -\frac{y}{x}$$

- `p1:=plot::vectorfield([1,-y/x],x=-10..10,y=-10..10)`

plot::Group()

- `p2:=plot::Scene(p1,Scaling=Constrained,Title="dy/dx=-y/x")`

plot::Group()

- `plot(p2)`

output omitted