Math 173 - Quiz 2

February 7, 2011

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find a set of parametric equations for the line of intersection of the two planes $x + 3y + 2z = 6$ and $2x - 2y + 3z = 7$.

$\vec{N}_1 = \hat{i} + 3\hat{j} + 2\hat{k}$

$\vec{N}_2 = 2\hat{i} - 2\hat{j} + 3\hat{k}$

$$\vec{N}_1 \times \vec{N}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 2 \\ 2 & -2 & 3 \end{vmatrix}$$

$$= 13\hat{i} + \hat{j} - 8\hat{k}$$

POINT ON BOTH PLANES ?

$y = 0 \Rightarrow \begin{array}{l} x + 2z = 6 \\ 2x + 3z = 7 \end{array} \Rightarrow \begin{array}{r} 2x + 4z = 12 \\ 2x + 3z = 7 \\ \hline z = 5 \end{array}$

$x = -4$

POINT $(-4, 0, 5)$

LINE

$$\boxed{\begin{aligned} x &= -4 + 13t \\ y &= t \\ z &= 5 - 8t \end{aligned}}$$

2. (2 points) Find an equation of the plane passing through the points $(1, 1, 1)$, $(0, -2, 3)$, and $(4, 1, -3)$.

$P\ (1, 1, 1)$

$Q\ (0, -2, 3)$

$R\ (4, 1, -3)$

$\vec{PQ} = -\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{PR} = 3\hat{i} - 4\hat{k}$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 2 \\ 3 & 0 & -4 \end{vmatrix}$$

$$= \underbrace{12\hat{i} + 2\hat{j} + 9\hat{k}}_{\text{NORMAL VECTOR } \vec{N}}$$

PLANE IS

$$12(x-1) + 2(y-1) + 9(z-1) = 0$$

OR $\boxed{12x + 2y + 9z = 23}$

3. (3 points) Find a point that lies exactly 5 units from the plane $2x + 2y - z = 8$.

THERE ARE LOTS OF WAYS TO DO THIS. HERE IS ONE APPROACH...

$\vec{N} = 2\hat{i} + 2\hat{j} - \hat{k}$

$|\vec{N}| = \sqrt{4+4+1} = 3$

SO $\vec{N}_1 = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$

IS A UNIT VECTOR.

POINT $(0,0,-8)$ IS ON THE LINE. SO A NORMAL LINE IS

$x = \frac{2}{3}t$

$y = \frac{2}{3}t$

$z = -8 - \frac{1}{3}t$

NOW TAKE $t = 5$.

$\left(\frac{10}{3}, \frac{10}{3}, -\frac{29}{3}\right)$ IS 5 UNITS FROM $(0,0,-8)$.

4. (2 points) See Theorem 11.14. Then find the distance between the point $\underbrace{(2,1,-3)}_{Q}$ and the line given by

$$\underbrace{\frac{x+2}{3} = \frac{y-8}{2} = z+5.}$$

DIRECTION VECTOR IS $\vec{u} = 3\hat{i} + 2\hat{j} + \hat{k}$

POINT IS $P(-2, 8, -5)$

$\vec{PQ} = 4\hat{i} - 7\hat{j} + 2\hat{k}$

$$\vec{PQ} \times \vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -7 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

$= -11\hat{i} + 2\hat{j} + 29\hat{k}$

$|\vec{PQ} \times \vec{u}| = \sqrt{(11)^2 + (2)^2 + (29)^2}$

$= \sqrt{966}$

$\vec{u} = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

$$\boxed{\text{DISTANCE} = \frac{\sqrt{966}}{\sqrt{14}} \approx 8.31}$$