Math 173 - Quiz 2

February 7, 2013

Name Key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (1 point) Find a set of symmetric equations for the line through $(3, 2, -1)$ and parallel to $\vec{v} = 5\hat{\imath} + 7\hat{\jmath} - 8\hat{k}$.

Point $(3, 2, -1)$

Direction $5\hat{\imath} + 7\hat{\jmath} - 8\hat{k}$ $\Rightarrow$ $\boxed{\dfrac{x-3}{5} = \dfrac{y-2}{7} = \dfrac{z+1}{-8}}$

2. (2 points) Find a set of parametric equations for the line segment connecting $(2, 0, -5)$ to $(3, 6, -3)$.

$\vec{V} = (3-2)\hat{\imath} + (6-0)\hat{\jmath} + (-3-(-5))\hat{k} = \hat{\imath} + 6\hat{\jmath} + 2\hat{k}$

Point $(2, 0, -5)$

$$\boxed{\begin{aligned} x &= 2 + t \\ y &= 6t \\ z &= -5 + 2t \end{aligned} \qquad 0 \le t \le 1}$$

3. (2 points) Find the angle between the planes.

$$7x - y - z = 8$$

$$3x + 5y + z = 2$$

$\vec{N}_1 = 7\hat{\imath} - \hat{\jmath} - \hat{k}$

$\vec{N}_2 = 3\hat{\imath} + 5\hat{\jmath} + \hat{k}$

$\vec{N}_1 \cdot \vec{N}_2 = 21 - 5 - 1 = 15$

$\|\vec{N}_1\| = \sqrt{49 + 1 + 1} = \sqrt{51}$

$\|\vec{N}_2\| = \sqrt{9 + 25 + 1} = \sqrt{35}$

$$\cos\theta = \frac{|\vec{N}_1 \cdot \vec{N}_2|}{\|\vec{N}_1\| \, \|\vec{N}_2\|} = \frac{15}{\sqrt{51}\sqrt{35}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{15}{\sqrt{51}\sqrt{35}}\right) \approx 69.2^\circ}$$

4. (5 points) Consider the two planes described by the equations given below.

$$-x + 2y - 3z = 6$$

$$2x - 4y + 6z = 4$$

(a) Show that the planes are parallel.

NORMAL VECTOR FOR 1ST PLANE $= \vec{N}_1 = -\hat{i} + 2\hat{j} - 3\hat{k}$

NORMAL VECTOR FOR 2ND PLANE $= \vec{N}_2 = 2\hat{i} - 4\hat{j} + 6\hat{k}$

$\vec{N}_2 = -2\vec{N}_1 \Rightarrow$ NORMAL VECTORS ARE PARALLEL

$\Rightarrow$ PLANES ARE PARALLEL.

(b) Find the distance between the planes.

POINT IN 2ND PLANE: $(0,-1,0)$

PLANE 1 : $-x + 2y - 3z = 6$

$$\text{DISTANCE} = \frac{|(-1)(0) + 2(-1) - 3(0) - 6|}{\sqrt{(-1)^2 + (2)^2 + (-3)^2}} = \boxed{\frac{8}{\sqrt{14}}}$$

(c) Choose any two points on one plane and a single point on the other plane. Then find an equation for the plane passing through your points.

TWO POINTS ON 1ST PLANE: $P(0,3,0)$ & $Q(0,0,-2)$

ONE POINT ON 2ND PLANE: $R(2,0,0)$

$\vec{PQ} = -3\hat{j} - 2\hat{k}$

$\vec{PR} = 2\hat{i} - 3\hat{j}$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -2 \\ 2 & -3 & 0 \end{vmatrix} = -6\hat{i} - 4\hat{j} + 6\hat{k}$$

Using $(0,3,0)$, my PLANE IS

$$-6(x-0) - 4(y-3) + 6(z-0) = 0$$

OR $\boxed{-6x - 4y + 6z = -12}$