Math 173 - Quiz 4

February 27, 2014

Name key

Score

Show each step to receive full credit. Supply explanations when necessary.

1. (4 points) Given the following information, find $\vec{r}(t)$.

$$\vec{r}''(t) = -4\cos t\,\hat{\jmath} - 3\sin t\,\hat{k}, \quad \vec{r}'(0) = \hat{\imath} + 3\hat{k}, \quad \vec{r}(0) = 2\hat{\imath} + 4\hat{\jmath}$$

$$\vec{r}''(t) = -4\cos t\,\hat{j} - 3\sin t\,\hat{k}$$

$$\Rightarrow \vec{r}'(t) = c_1\hat{\imath} + (-4\sin t + c_2)\hat{j} + (3\cos t + c_3)\hat{k}$$

$$\vec{r}'(0) = c_1\hat{\imath} + c_2\hat{j} + (3+c_3)\hat{k} = \hat{\imath} + 3\hat{k} \Rightarrow c_1 = 1,\ c_2 = 0,\ c_3 = 0$$

$$\vec{r}'(t) = \hat{\imath} - 4\sin t\,\hat{j} + 3\cos t\,\hat{k}$$

$$\Rightarrow \vec{r}(t) = (t + c_4)\hat{\imath} + (4\cos t + c_5)\hat{j} + (3\sin t + c_6)\hat{k}$$

$$\vec{r}(0) = c_4\hat{\imath} + (4 + c_5)\hat{j} + c_6\hat{k} = 2\hat{\imath} + 4\hat{j} \Rightarrow c_4 = 2,\ c_5 = 0,\ c_6 = 0$$

$$\boxed{\vec{r}(t) = (t+2)\hat{\imath} + 4\cos t\,\hat{j} + 3\sin t\,\hat{k}}$$

2. (4 points) Let $\vec{u}(t) = t\hat{\imath} + 3t\hat{\jmath} + t^2\hat{k}$ and $\vec{w}(t) = 4t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}$. Compute $\frac{d}{dt}[\vec{u}(t) \times \vec{w}(t)]$.

$$\vec{u} \times \vec{w} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ t & 3t & t^2 \\ 4t & t^2 & t^3 \end{vmatrix} = \hat{\imath}(3t^4 - t^4) - \hat{\jmath}(t^4 - 4t^3) + \hat{k}(t^3 - 12t^2)$$

$$= 2t^4\hat{\imath} + (4t^3 - t^4)\hat{\jmath} + (t^3 - 12t^2)\hat{k}$$

$$\frac{d}{dt}[\vec{u} \times \vec{w}] = \boxed{8t^3\hat{\imath} + (12t^2 - 4t^3)\hat{\jmath} + (3t^2 - 24t)\hat{k}}$$

3. (2 points) Describe the graphs of the vector-valued functions $\vec{r_1}(t) = t\hat{\imath} + t^2\hat{\jmath}$ and $\vec{r_2}(t) = t\hat{\imath} + t^2\hat{\jmath} + t\hat{k}$.

THE GRAPH OF $\vec{r}_1(t)$ IS THE GRAPH OF $y = x^2$. THIS IS A 2D GRAPH IN THE XY-PLANE.

THE GRAPH OF $\vec{r}_2(t)$ IS A 3D CURVE. IT IS THE GRAPH OF $y = x^2$ (IN THE PLANE) BUT TILTED OUT OF THE XY-PLANE SO THAT IT LIES IN THE $X = Z$ PLANE.