Math 173 - Quiz 2

January 29, 2015

Name key

Score ______

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the area of the parallelogram determined by the vectors $\vec{u} = 3\hat{\imath} - 5\hat{\jmath} - 2\hat{k}$ and $\vec{v} = \hat{\imath} - 4\hat{\jmath} + 2\hat{k}$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & -2 \\ 1 & -4 & 2 \end{vmatrix} = \hat{i}(-18) - \hat{j}(8) + \hat{k}(-7)$$
$$= -18\hat{i} - 8\hat{j} - 7\hat{k}$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{(-18)^2 + (-8)^2 + (-7)^2} = \sqrt{437}$$

Area $= \|\vec{u} \times \vec{v}\| = \sqrt{437} \approx 20.9$

2. (2 points) Find a nonzero vector whose cross product with $\vec{w} = -\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ is the zero vector.

ANY VECTOR PARALLEL TO $\vec{\omega}$ WOULD WORK.

FOR EXAMPLE, $\vec{\omega} \times \vec{\omega} = \vec{0}$.

3. (3 points) Refer to the vectors $\vec{u}$ and $\vec{v}$ in problem #1. Find the projection of $\vec{u}$ onto $\vec{v}$.

$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v} = \frac{3 + 20 - 4}{1 + 16 + 4} \vec{v} = \frac{19}{21} \vec{v}$$
$$= \frac{19}{21}(\hat{i} - 4\hat{j} + 2\hat{k})$$

4. (2 points) If $\vec{u}$ is a nonzero vector such that $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$, must it be true that $\vec{v} = \vec{w}$? Explain your reasoning.

No, $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{\omega} \Rightarrow \vec{u} \cdot (\vec{v} - \vec{\omega}) = 0$

NOW, LET $\vec{X}$ BE $\underline{\underline{\text{ANY}}}$ NONZERO VECTOR ORTHOGONAL TO $\vec{u}$ AND LET $\vec{V}$ AND $\vec{\omega}$ BE DISTINCT VECTORS SUCH THAT $\vec{X} = \vec{V} - \vec{\omega}$.

FOR EXAMPLE,

$\vec{u} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{X} = -3\hat{i} + \hat{k}$

$\vec{V} = \hat{j} + \hat{k}$

$\vec{\omega} = 3\hat{i} + \hat{j}$

$\vec{u} \cdot \vec{V} = \vec{u} \cdot \vec{\omega} = 5$