

# Math 173 - Quiz 7

March 12, 2015

Name key \_\_\_\_\_  
Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) In 1897 Thomson conducted a landmark experiment in modern physics when he measured the charge-to-mass ratio of an electron. The currently accepted value of the electron's charge is  $1.602176487 \times 10^{-19}$  C with an error of  $\pm 0.000000040 \times 10^{-19}$  C. The electron's mass is  $9.10938215 \times 10^{-31}$  kg, with an error of  $\pm 0.00000045 \times 10^{-31}$  kg. Compute the electron's charge-to-mass ratio and use differentials to approximate the error.

$$\text{CHARGE-TO-MASS RATIO} = z = \frac{q}{m} \Rightarrow dz = \frac{1}{m} dq - \frac{q}{m^2} dm$$

$$\Rightarrow \Delta z \approx \frac{1}{m} \Delta q - \frac{q}{m^2} \Delta m$$

USING THE NUMBERS ABOVE, we get  $\Delta z \approx -4297$

$$\frac{q}{m} = 1.758820149 \times 10^{-19} \pm \underbrace{4297}_{0.00000004297 \times 10^{-19}}$$

2. (3 points) Use differentials to approximate  $\underbrace{\sin(1.05^2 + 0.95^2) - \sin(1^2 + 1^2)}$ .

$$z = f(x,y) = \sin(x^2 + y^2)$$

$$x=y=1 \text{ AND } \Delta x = 0.05, \Delta y = -0.05$$

$$\Delta z \approx 2x \cos(x^2 + y^2) \Delta x + 2y \cos(x^2 + y^2) \Delta y$$

$$\Delta z \approx 2 \cos(2)(0.05) + 2 \cos(2)(-0.05) = \boxed{0}$$

3. (4 points) Use the definition of differentiability to show that  $f(x, y) = 8x^2 - 4xy + y^3$  is differentiable on  $\mathbb{R}^2$ .

$$f_x(x, y) = 16x - 4y$$

$$f_y(x, y) = -4x + 3y^2$$

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y) =$$

$$8(x + \Delta x)^2 - 4(x + \Delta x)(y + \Delta y) + (y + \Delta y)^3 - 8x^2 + 4xy - y^3 =$$

$$\cancel{8x^2} + \cancel{16x\Delta x} + \underline{\cancel{8\Delta x^2}} - \cancel{4xy} - \cancel{4y\Delta x} - \cancel{4x\Delta y} - \cancel{4\Delta x\Delta y} + \cancel{y^3} + \underline{\cancel{3y^2\Delta y}} + \underline{\cancel{3y\Delta y^2}} + \cancel{\Delta y^3} - \cancel{8x^2} + \cancel{4xy} - \cancel{y^3} =$$

$$(16x - 4y)\Delta x + (-4x + 3y^2)\Delta y + (8\Delta x - 4\Delta y)\Delta x + (3y\Delta y + \Delta y^2)\Delta y$$

$$\begin{matrix} \uparrow \\ f_x(x, y) \end{matrix}$$

$$\begin{matrix} \uparrow \\ f_y(x, y) \end{matrix}$$

$$\begin{matrix} \uparrow \\ \epsilon_1 \end{matrix}$$

$$\begin{matrix} \uparrow \\ \epsilon_2 \end{matrix}$$

AND  $(\epsilon_1, \epsilon_2) \rightarrow (0, 0)$  AS

$$(\Delta x, \Delta y) \rightarrow (0, 0).$$

THIS IS TRUE FOR ALL  $(x, y)$ .

$f$  IS DIFFERENTIABLE EVERYWHERE.