Math 173 - Quiz 9

April 9, 2015

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Find the absolute extreme values of $f(x, y) = xy^2 - 4x$ subject to $x^2 + y^2 = 4$.

$\vec{\nabla} f(x,y) = (y^2-4)\hat{\imath} + (2xy)\hat{\jmath}$

$\vec{\nabla} g(x,y) = 2x\hat{\imath} + 2y\hat{\jmath}$

$y^2 - 4 = \lambda 2x$

$2xy = \lambda 2y \Rightarrow \lambda = x$ OR $y = 0$

$x^2 + y^2 = 4$

$y = 0$:

$x^2 = 4$

$x = \pm 2$

$\lambda = x$:

$y^2 - 4 = 2x^2$

$\Rightarrow y^2 = 2x^2 + 4$

$x^2 + 2x^2 + 4 = 4 \Rightarrow x = 0 \Rightarrow y = \pm 2$

CRITICAL POINTS ARE

$(2,0), (-2,0), (0,2), (0,-2)$

$f(2,0) = -8 \leftarrow$ MIN

$f(-2,0) = 8 \leftarrow$ MAX

$f(0,2) = 0$

$f(0,-2) = 0$

2. (5 points) Find the absolute minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x + y - z = 3$. Explain why there is no maximum value.

$\vec{\nabla} f(x,y,z) = 2x\hat{\imath} + 2y\hat{\jmath} + 2z\hat{k}$

$\vec{\nabla} g(x,y,z) = \hat{\imath} + \hat{\jmath} - \hat{k}$

$2x = \lambda$

$2y = \lambda$

$2z = -\lambda$

$x + y - z = 3$

$\Downarrow$

$\lambda + \lambda + \lambda = 6$

$\lambda = 2 \Rightarrow x = 1, y = 1, z = -1$

$f(1,1,-1) = 3$ IS A MIN

THERE IS NO MAX. f MEASURES THE SQUARE OF THE DISTANCE FROM (x,y,z) TO $(0,0,0)$. THERE IS NO POINT (x,y,z) ON THE PLANE $x+y-z=3$ THAT IS FARTHEST FROM THE ORIGIN.