# Math 173 - Test 1

February 12, 2015

Name ______________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Find a vector of magnitude 7 that has the direction from $P\ (2,5,8)$ to $Q\ (-1,3,4)$.

2. (6 points) Let $\vec{u} = -5\hat{\imath} + 4\hat{\jmath} - \hat{k}$.

   (a) Find a vector, different from $\vec{u}$, that is parallel to $\vec{u}$. Give a one-sentence explanation of how you know.

   (b) Find a nonzero vector that is orthogonal to $\vec{u}$. Give a one-sentence explanation of how you know.

3. (2 points) Without actually computing the cross product, use the right-hand rule to determine $\hat{k} \times \hat{\imath}$.

4. (6 points) Find a set of symmetric equations for the line passing through $(2, 1, 5)$ and parallel to the line with parametric equations $x = 3 + 2t$, $y = -6 - t$, $z = 7 - 5t$.

5. (9 points) In the following problems, the force vectors $\vec{F_1}$ and $\vec{F_2}$ are 2D vectors in the $xy$-plane.

   (a) The force $\vec{F_1}$ has magnitude 3 and makes a $225°$ angle with the positive $x$-axis. Find the component form of $\vec{F_1}$.

   (b) The force $\vec{F_2}$ has component form $\vec{F_2} = 5\sqrt{3}\hat{\imath} + 10\hat{\jmath}$. Compute the resultant vector $\vec{F} = \vec{F_1} + \vec{F_2}$.

   (c) What angle does $\vec{F}$ make with the positive $x$-axis?

6. (4 points) The angle between $\vec{u}$ and $\vec{v}$ is obtuse. What can be said about $\vec{u} \cdot \vec{v}$? Briefly explain.

7. (6 points) Using the parameter $x = t$, find a vector-valued function (i.e., a set of parametric equations) that describes the curve of intersection of the surfaces $z = x^2 + y^2$ and $x + y = 0$.

8. (6 points) Find the distance from the point $P\ (7, 1, -3)$ to the plane $4x - 2y + z = 5$.

9. (6 points) Find the projection of $\vec{w} = \hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ onto $\vec{u} = 7\hat{\imath} + 4\hat{k}$.

10. (8 points) Find a set of parametric equations for a line in the plane $5x - 9y - 8z = 5$.

11. (7 points) Find the vector-valued function $\vec{r}(t)$ such that

$$\vec{r}\,'(t) = te^{-t^2}\hat{\imath} - e^{-t}\hat{\jmath} + \hat{k}; \quad \vec{r}(0) = \frac{1}{2}\hat{\imath} - \hat{\jmath} + 2\hat{k}.$$

12. (6 points) Sketch the graph of the vector-valued function $\vec{r}(t) = t^3\hat{\imath} + t\hat{\jmath}$. Draw arrows on your graph to indicate the curve's orientation.

13. (10 points) Let $\vec{r}(t) = 2\cos t\,\hat{\imath} - e^t\hat{\jmath} + t^2\hat{k}$.

(a) Let $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$. Compute $\vec{T}(t)$.

(b) If you were forced to compute $\vec{T}(t) \cdot \vec{T}\,'(t)$, what would you find? Explain.

14. (10 points) Let $\vec{u} = -2\hat{\imath} + 9\hat{\jmath} + \hat{k}$ and $\vec{v} = \hat{\imath} - \hat{\jmath} + 4\hat{k}$.

(a) Find a vector orthogonal to both $\vec{u}$ and $\vec{v}$.

(b) Find an equation of the plane passing through $(5, 0, 3)$ with normal vector is $\vec{u} \times \vec{v}$.

15. (8 points) A crystal structure has the form of a parallelepiped determined by the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b} = 3\hat{\jmath} + 5\hat{k}$, and $\vec{c} = -4\hat{\imath} + 2\hat{\jmath} + \hat{k}$, where distances are measured in micrometers. Find the volume of the parallelepiped.