

Math 173 - Test 2
March 26, 2015

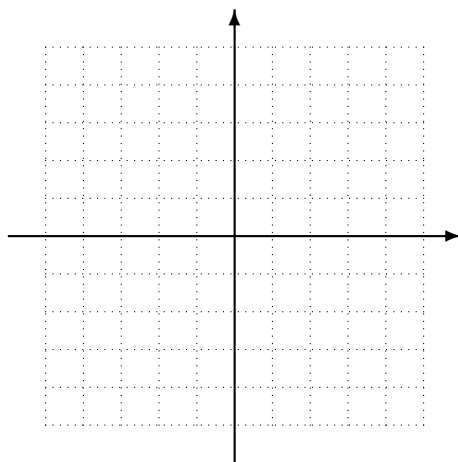
Name _____

Score _____

Show all work. Supply explanations when necessary.

1. (8 points) Suppose the equation $xyz + 3xy^2 + xe^{yz} - 4x = 0$ implicitly defines z as a function of x and y . Find $\partial z / \partial y$ at the point $(1, 1, 0)$.

2. (4 points) Sketch a curve whose curvature function is the constant function $\kappa = 1/3$. Briefly explain.



3. (10 points) Consider the function $P(x, y) = \ln(4 + x^2 + y^2)$.
- (a) Find the directional derivative of P at the point $(-1, 2)$ in the direction of $\vec{w} = 2\hat{i} + \hat{j}$.
- (b) At the point $(-1, 2)$, what is the direction of steepest descent (maximum decrease) of P ?
4. (10 points) The *body mass index* (BMI) for an adult human is given by $B = 703w/h^2$, where w is weight in pounds and h is height in inches. Suppose you weigh 190 lbs and your height is 70 in. Your weight and height measurements have possible errors $\Delta w = \pm 1.5$ lbs and $\Delta h = \pm 0.5$ in. Use differentials to estimate the error in your BMI.

5. (6 points) Suppose z is a function of x, y ; and x, y are functions of t, u, v . Write the chain rule formulas for $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial u}$.

6. (10 points) Evaluate each limit or show that it does not exist.

(a) $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 + xy - 2y^2}{2x^2 - xy - y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$

7. (12 points) At time $t = 0$, a baseball is hit 3 ft above the ground at an angle of 45° with a speed of $80\sqrt{2}$ ft/s. Neglect all forces other than gravity. (Use $g = 32$ ft/s².)

(a) Find the vector-valued functions that give the position and velocity of the ball at time t .

(b) What is the maximum height of the ball?

(c) Will the ball clear a 20-ft fence that is 380 ft downrange?

8. (12 points) Let $\vec{r}(t) = \frac{t^2}{2}\hat{i} + (4 - 3t)\hat{j} + 2\hat{k}$. Find the principal unit normal vector at the point where $t = 0$.

9. (10 points) Consider the function $g(x, y) = \ln(x^2 + y)$.

(a) What is the domain of g ?

(b) Discuss the continuity of g .

(c) Sketch the level curve $g(x, y) = 0$.

(d) Compute the mixed partial derivative g_{xy} .

(e) Without actually computing the mixed partial derivative g_{yx} , would you expect it to be equal to g_{xy} ? Explain.

10. (8 points) For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, let $f(x) = \ln(\cos x)$. Find the curvature function $\kappa(x)$.

11. (4 points) Consider the function $f(x, y) = \frac{xy}{x^2 + y^2}$. Do you expect that f is a differentiable function? Where? How do you know?

12. (6 points) Let $g(x, y, z) = 2x^2yz - \cos(xy) + z^3e^{-x}$. Find $\vec{\nabla}g(x, y, z)$.