

Math 173 - Final Exam

May 11, 2015

Name _____

Score _____

Show all work. Supply explanations when necessary. **Do any 15 of the following 16 problems. Circle the number of the problem you DO NOT want graded.** Each problem is worth 10 points.

1. Find a vector of magnitude 7 that is orthogonal to $3\hat{i} - 2\hat{j} + 5\hat{k}$.

2. Suppose that $w = 3xy + yz$ and that x , y , and z are functions of u and v such that

$$x = \ln u + \cos v, \quad y = 1 + u \sin v, \quad z = uv.$$

Use the chain rule to find $\partial w / \partial u$ at $(u, v) = (1, \pi)$.

3. Sketch the region R whose area is given by the iterated integral. Then reverse the order of integration and evaluate the new iterated integral by hand.

$$\int_0^4 \int_0^{x/2} dy \, dx + \int_4^6 \int_0^{6-x} dy \, dx$$

4. Find a set of parametric equations for the line tangent to the graph of $\vec{r}(t)$ at the point $(e, 0, 2)$.

$$\vec{r}(t) = te^t \hat{i} + \sin(\pi t) \hat{j} + \sqrt{3+t^2} \hat{k}$$

5. A batter hits a baseball 3 ft above the ground toward the center field fence, which is 10 ft high and 400 ft from home plate. The ball leaves the bat with a speed of 115 ft/sec at an angle of 51° above the horizontal. Does the ball clear the fence? (Use $g = 32 \text{ ft/sec}^2$.)
6. Use Lagrange multipliers to find the extreme values of $f(x, y) = \frac{1}{3}x^3 + y^2$ on the unit circle $x^2 + y^2 = 1$.

7. A thin plate is bounded by the graphs of $y = x^2$ and $y = 2 - x$. The density of the plate at the point (x, y) is given by $\rho(x, y) = 5 + xy$. Set up the iterated integrals that are required to determine the center of mass of the plate. Then tell how the values of those iterated integrals would be used to calculate the center of mass. Do not evaluate the integrals.
8. A plane passes through the points $P(2, 1, 3)$, $Q(-7, 6, -1)$ and $R(3, 0, -1)$. Find a set of symmetric equations for the line normal to the plane and passing through $(2, 1, 3)$.

9. Evaluate the limit or show that it does not exist.

(a) $\lim_{(x,y) \rightarrow (1,1)} \frac{3x - 3y}{y^2 - x^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{2x - y^2}{2x^2 + y}$

10. Find and classify all relative extreme values of the function $f(x, y)$.

$$f(x, y) = x^2 + xy + 2y^2 + x - 3y + 10$$

11. A surface is described by the equation $xy^2 + 3x - z^2 = 8$. Find an equation of the plane tangent to the surface at the point $(1, -3, 2)$.

12. Let $\vec{r}(t) = 2t\hat{i} + t^2\hat{j} + 2t\hat{k}$. Find the principal unit normal vector at $t = 0$. (Hint: Find $\hat{T}'(0)$ and then normalize it.)

13. Let \vec{u} be the vector from $(6, 3, 1)$ to $(8, 0, 4)$. Let \vec{v} be the vector in the xy -plane with magnitude 4 that makes an angle of 30° with the positive x -axis. Find $\text{proj}_{\vec{u}} \vec{v}$.
14. Use a triple integral to find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x + y + 4z = 4$.

15. Use a double integral to find the area of the polar region inside the circle $r = 2 \cos \theta$ and outside the circle $r = 1$.
16. Consider the function $f(x, y, z) = xy^3 - \frac{\sqrt{x}}{z^2}$. Find the directional derivative of f at $(4, 1, 2)$ in the direction of $\hat{i} + 2\hat{j} - \hat{k}$.