Math 173 - Quiz 11

May 3, 2018

Name Key

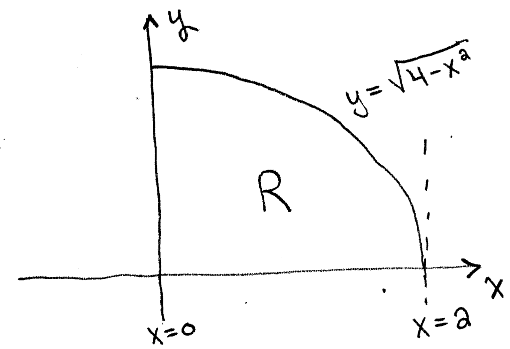
Score

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) The space region D lies in the 1st octant bounded by the coordinate planes and the sufaces $x^2 + y^2 = 4$ and $z = x + y$. Find the average value of

$$f(x, y, z) = x + y^2 + z^3$$

on D. Use your calculator or computer algebra system to evaluate all required integrals.



THE SURFACE $z = x+y$ LIES ABOVE R AND PASSES THROUGH $(0,0,0)$

VOLUME OF D =

$$\iiint_D dV = \iint_R \int_{z=0}^{z=x+y} dz\, dA$$

$$= \int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{4-x^2}} \int_{z=0}^{z=x+y} dz\, dy\, dx$$

$$= \frac{16}{3}$$

AVERAGE VALUE =

$$\frac{1}{16/3} \int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{4-x^2}} \int_{z=0}^{z=x+y} (x + y^2 + z^3)\, dz\, dy\, dx$$

$$= \boxed{\frac{103}{40} + \frac{9\pi}{16} \approx 4.34}$$

MATHEMATICA OUTPUT ATTACHED.

In[24]:= $\text{Int} = \int_0^2 \left(\int_0^{\sqrt{4-x^2}} \left(\int_0^{x+y} \left(x + y^2 + z^3\right) dz \right) dy \right) dx$

Out[24]= $\frac{206}{15} + 3\pi$

In[25]:= $V = \int_0^2 \left(\int_0^{\sqrt{4-x^2}} \left(\int_0^{x+y} 1\, dz \right) dy \right) dx$

Out[25]= $\frac{16}{3}$

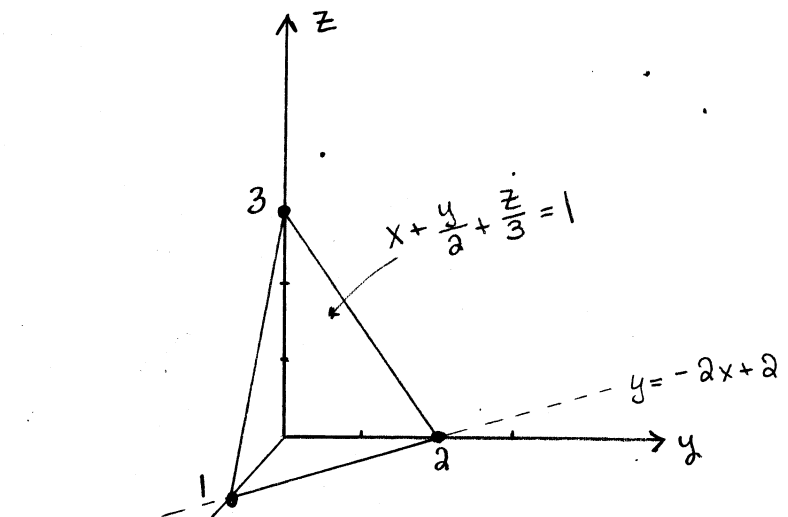
In[26]:= Int / V

Out[26]= $\frac{3}{16} \left(\frac{206}{15} + 3\pi \right)$

In[27]:= N[Int / V]

Out[27]= 4.34215

2. (6 points) Let T be the solid tetrahedron in the 1st octant whose vertices are $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$. The density of the tetrahedron at (x,y,z) is given by $\rho(x,y,z) = x+y+z$. Find the center of mass. Use your calculator or computer algebra system to evaluate all required integrals.



$$M = \int_{x=0}^{x=1} \int_{y=0}^{y=2-2x} \int_{z=0}^{z=3(1-x-\frac{y}{2})} (x+y+z)\, dz\, dy\, dx = \frac{3}{2}$$

$$M_{yz} = \int_{x=0}^{x=1} \int_{y=0}^{y=2-2x} \int_{z=0}^{z=3(1-x-\frac{y}{2})} x \cdot (x+y+z)\, dz\, dy\, dx = \frac{7}{20}$$

$$M_{xz} = \int_{x=0}^{x=1} \int_{y=0}^{y=2-2x} \int_{z=0}^{z=3(1-x-\frac{y}{2})} y \cdot (x+y+z)\, dz\, dy\, dx = \frac{4}{5}$$

$$M_{xy} = \int_{x=0}^{x=1} \int_{y=0}^{y=2-2x} \int_{z=0}^{z=3(1-x-\frac{y}{2})} z \cdot (x+y+z)\, dz\, dy\, dx = \frac{27}{20}$$

$$C.M. = \left(\frac{M_{yz}}{M}, \frac{M_{xz}}{M}, \frac{M_{xy}}{M} \right) = \boxed{\left(\frac{7}{30}, \frac{8}{15}, \frac{9}{10} \right)}$$

MATHEMATICA OUTPUT IS ATTACHED.

In[13]:= $M = \int_0^1 \left(\int_0^{2-2x} \left(\int_0^{3\left(1-x-\frac{y}{2}\right)} (x+y+z)\, dz\right) dy\right) dx$

Out[13]= $\frac{3}{2}$

In[14]:= $Myz = \int_0^1 \left(\int_0^{2-2x} \left(\int_0^{3\left(1-x-\frac{y}{2}\right)} x * (x+y+z)\, dz\right) dy\right) dx$

Out[14]= $\frac{7}{20}$

In[15]:= $Mxz = \int_0^1 \left(\int_0^{2-2x} \left(\int_0^{3\left(1-x-\frac{y}{2}\right)} y * (x+y+z)\, dz\right) dy\right) dx$

Out[15]= $\frac{4}{5}$

In[16]:= $Mxy = \int_0^1 \left(\int_0^{2-2x} \left(\int_0^{3\left(1-x-\frac{y}{2}\right)} z * (x+y+z)\, dz\right) dy\right) dx$

Out[16]= $\frac{27}{20}$

In[17]:= Myz / M

Out[17]= $\frac{7}{30}$

In[18]:= Mxz / M

Out[18]= $\frac{8}{15}$

In[19]:= Mxy / M

Out[19]= $\frac{9}{10}$