

# Math 173 - Quiz 3

February 1, 2018

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (1 point) Consider the planes described by the equations given below.

$$2x - 5y + z = 8$$

$$x + 2y + 3z = 10$$

How do you know these are NOT parallel?

$$\vec{n}_1 = 2\hat{i} - 5\hat{j} + \hat{k}$$

$$\vec{n}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$$

} THE PLANES' NORMAL VECTORS ARE NOT PARALLEL.

2. (3 points) Find the angle between the planes described above.

$$\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{|2 - 10 + 3|}{\sqrt{30} \sqrt{14}} = \frac{5}{\sqrt{420}}$$

$$\theta = \cos^{-1} \left( \frac{5}{\sqrt{420}} \right) \approx 75.9^\circ$$

3. (3 points) Find the area of the triangle with vertices  $P(1, 2, 3)$ ,  $Q(-4, 0, -2)$ , and  $R(3, -1, -2)$ .

$$\vec{PQ} = -5\hat{i} - 2\hat{j} - 5\hat{k}$$

$$\vec{PR} = 2\hat{i} - 3\hat{j} - 5\hat{k}$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -2 & -5 \\ 2 & -3 & -5 \end{vmatrix} = -5\hat{i} - 35\hat{j} + 19\hat{k}$$

$$\text{Area} = \frac{1}{2} \|\vec{PQ} \times \vec{PR}\| = \frac{1}{2} \sqrt{25 + 1225 + 361} = \frac{1}{2} \sqrt{1611} \approx 20.07$$

4. (2 points) Referring to the triangle in problem #3, find symmetric equations for the line passing through  $P$  and normal to the triangle.

$$\frac{x-1}{-5} = \frac{y-2}{-35} = \frac{z-3}{19}$$

5. (1 point) Referring to the triangle in problem #3, find an equation for the plane of the triangle.

$$-5(x-1) - 35(y-2) + 19(z-3) = 0$$

$$-5x - 35y + 19z = -5 - 70 + 57$$

$$-5x - 35y + 19z = -18$$