Math 173 - Quiz 4

February 8, 2018

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the distance from the point $(5, 2, -1)$ to the plane described by the equation $2x - 3y + 5z = 8$.

$$\text{DISTANCE} = \frac{|2(5) - 3(2) + 5(-1) - 8|}{\sqrt{(2)^2 + (-3)^2 + (5)^2}} = \frac{9}{\sqrt{38}} \approx 1.46$$

2. (3 points) Find the distance from the point $(5, 2, -1)$ to the line described by the symmetric equations

$$\frac{x-1}{4} = -y = \frac{z+7}{6}.$$

$P(1, 0, -7)$

$Q(5, 2, -1)$

$\vec{PQ} = 4\hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{v} = 4\hat{i} - \hat{j} + 6\hat{k}$

$$\vec{PQ} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 6 \\ 4 & -1 & 6 \end{vmatrix} = 18\hat{i} - 12\hat{k}$$

$$\|\vec{PQ} \times \vec{v}\| = \sqrt{(18)^2 + (-12)^2} = \sqrt{468}$$

$$\text{DISTANCE} = \frac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|} = \frac{\sqrt{468}}{\sqrt{16+1+36}} = \sqrt{\frac{468}{53}} \approx 2.97$$

3. (2 points) Find $\vec{r}(t)$ if $\frac{d}{dt}\vec{r}(t) = \frac{2}{1+t^2}\hat{i} + \frac{1}{t+1}\hat{j}$ and $\vec{r}(0) = 3\hat{i} - 2\hat{j}$.

$$\vec{r}(t) = (2\tan^{-1} t + C_1)\hat{i} + (\ln|t+1| + C_2)\hat{j}$$

$$\vec{r}(0) = 3\hat{i} - 2\hat{j} = C_1\hat{i} + C_2\hat{j} \Rightarrow C_1 = 3,\ C_2 = -2$$

$$\vec{r}(t) = (2\tan^{-1} t + 3)\hat{i} + (\ln|t+1| - 2)\hat{j}$$

4. (2 points) State the name of the quadric surface described by each equation.

$4x^2 + 6y^2 = z$ — PARABOLOID.

$x^2 - 3y^2 + 7 = z^2$ — $3y^2 + z^2 = x^2 + 7$ — HYPERBOLOID OF ONE SHEET.