

Math 173 - Quiz 5

February 22, 2018

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (6 points) Find the principal unit normal vector at $t = 0$.

$$\vec{r}'(t) = \sqrt{a} \hat{i} + e^t \hat{j} - e^{-t} \hat{k}$$

$$\begin{aligned} \|\vec{r}'(t)\| &= \sqrt{a + e^{2t} + e^{-2t}} \\ &= \sqrt{(e^t + e^{-t})^2 a} \\ &= e^t + e^{-t} \end{aligned}$$

$$\hat{T}(t) = \frac{\sqrt{a} \hat{i} + e^t \hat{j} - e^{-t} \hat{k}}{e^t + e^{-t}}$$

$$\vec{r}(t) = \sqrt{2}t\hat{i} + e^t\hat{j} + e^{-t}\hat{k}$$

$$\hat{T}'(t) = \frac{(e^t + e^{-t})(e^t \hat{j} + e^{-t} \hat{k}) - (\sqrt{a} \hat{i} + e^t \hat{j} - e^{-t} \hat{k})(e^t - e^{-t})}{(e^t + e^{-t})^2}$$

$$\hat{T}'(0) = \frac{a(\hat{j} + \hat{k})}{4} = \frac{1}{a}(\hat{j} + \hat{k})$$

$$\|\hat{T}'(0)\| = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{a}}$$

$$\hat{N}(0) = \frac{\hat{T}'(0)}{\|\hat{T}'(0)\|} = \sqrt{\frac{1}{a}}(\hat{j} + \hat{k})$$

2. (4 points) A projectile is launched from the ground at 50 ft/s at an angle of 30° with the horizontal. When does the projectile hit the ground? Set up the definite integral that gives the length of the path of the projectile.

$$\begin{aligned} \vec{r}(t) &= 50 \cos 30^\circ t \hat{i} + (-16t^2 + 50 \sin 30^\circ t) \hat{j} \\ &= 25\sqrt{3}t \hat{i} + (-16t^2 + 25t) \hat{j} \end{aligned}$$

$$-16t^2 + 25t = 0 \Rightarrow t = 0 \text{ or } t = \frac{25}{16}$$

$$\text{Length} = \int_0^{25/16} \sqrt{(25\sqrt{3})^2 + (-32t + 25)^2} dt$$