Math 173 - Quiz 7

March 8, 2017

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (2.5 points) An object is thrown straight upward so that its height at time t is given by
$$h(t) = -\frac{1}{2}gt^2 + v_0t.$$
Suppose $g = (32.26 \pm 0.15)\ \text{ft/s}^2$ and $v_0 = (92.44 \pm 0.12)\ \text{ft/s}$. Determine the height after 5 seconds and use differentials to estimate the error in the computed value. Assume that the error in t is 0.01 s.

$$h(5) \approx -\frac{1}{2}(32.26)(25) + (92.44)(5) = 58.95\ \text{FT}$$

$$-0.15 \le \Delta g \le 0.15$$
$$-0.12 \le \Delta v_0 \le 0.12$$
$$-0.01 \le \Delta t \le 0.1$$

$$dh = -\frac{1}{2}t^2\,dg + t\,dv_0 + (-gt + v_0)\,dt$$

$$\Delta h \approx -\frac{1}{2}t^2\,\Delta g + t\,\Delta v_0 + (-gt + v_0)\,\Delta t$$

$$\Delta h \le -\frac{1}{2}(5)^2(-0.15) + (5)(0.12) + (-32.26(5) + 92.44)(-0.01)$$

$$= 1.875 + 0.6 + 0.6886 = 3.1636$$

$$\boxed{\text{HEIGHT} = (58.95 \pm 3.16)\ \text{FT}}$$

2. (2.5 points) Find the limit or show that it does not exist: $\displaystyle\lim_{(x,y)\to(1,4)} \frac{xy - 2x - y + 2}{x^2y + x^2 - y - 1}$ $\quad \frac{0}{0}$

$$\lim_{(x,y)\to(1,4)} \frac{(y-2)(x-1)}{(x^2-1)(y+1)} = \lim_{(x,y)\to(1,4)} \frac{y-2}{(x+1)(y+1)} = \frac{2}{2(5)}$$

$$= \boxed{\frac{1}{5}}$$

3. (2.5 points) Use differentials to approximate the quantity below.

$$\underbrace{\frac{1-(3.05)^2}{(5.95)^2} - \frac{1-3^2}{6^2}}$$

$f(x,y) = \dfrac{1-x^2}{y^2}$

$\Delta x = 0.05$

$\Delta y = -0.05$

$x = 3$

$y = 6$

$f(x+\Delta x, y+\Delta y) - f(x,y) = \Delta z$

$$\Delta z \approx \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y$$

$$= \frac{-2x}{y^2}\Delta x + \frac{-2(1-x^2)}{y^3}\Delta y$$

$$\Delta z \approx \frac{-2(3)}{36}(0.05) + \frac{-2(-8)}{216}(-0.05)$$

$$\approx -0.012$$

4. (2.5 points) Find the limit or show that it does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x-y^2}{2x^2+y}$

Along $x=0$:

$$\lim_{y\to 0} \frac{-y^2}{y} = \lim_{y\to 0}(-y) = 0$$

LIMIT DNE.

Along $y=x$:

$$\lim_{x\to 0} \frac{2x-x^2}{2x^2+x} = \lim_{x\to 0} \frac{2-x}{2x+1} = 2$$