Math 173 - Quiz 8

March 29, 2018

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Let $f(x, y, z) = xy^2z^4$. Find the maximum value of the directional derivative of f at the point $(2, 1, 1)$.

$$\vec{\nabla} f(x,y,z) = y^2z^4\,\hat{i} + 2xyz^4\,\hat{j} + 4xy^2z^3\,\hat{k}$$

$$\vec{\nabla} f(2,1,1) = \hat{i} + 4\hat{j} + 8\hat{k}$$

MAX VALUE OF DIRECTIONAL DERIV $= \|\vec{\nabla} f(2,1,1)\|$

$$= \sqrt{1+16+64} = \boxed{9}$$

2. (3 points) Find an equation of the plane tangent to the surface $x = y(2z - 3)$ at the point $(4, 4, 2)$.

$$\underbrace{x - 2yz + 3y}_{F(x,y,z)} = 0$$

OUR SURFACE IS THE LEVEL SURFACE $F(x,y,z) = 0$

$$\vec{\nabla} F(x,y,z) = \hat{i} + (-2z+3)\hat{j} + (-2y)\hat{k}$$

$$\vec{n} = \vec{\nabla} F(4,4,2) = \hat{i} - \hat{j} - 8\hat{k}$$

$$1(x-4) + (-1)(y-4) + (-8)(z-2) = 0$$

$$\boxed{x - y - 8z = -16}$$

3. (4 points) Find the linearization of $f(x, y) = x^2 y \sin(\pi xy)$ at the point $(1, 1)$. Then use your linearization to approximate $f(0.96, 1.02)$.

$$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$$

$$f_x(x,y) = 2xy \sin \pi xy + \pi x^2y^2 \cos \pi xy \qquad f_x(1,1) = -\pi$$

$$f_y(x,y) = x^2 \sin \pi xy + \pi x^3 y \cos \pi xy \qquad f_y(1,1) = -\pi$$

$$L(x,y) = 0 - \pi(x-1) - \pi(y-1)$$

$$= -\pi x - \pi y + 2\pi$$

$$f(0.96, 1.02) \approx L(0.96, 1.02) = -\pi(0.96) - \pi(1.02) + 2\pi$$

$$= -1.98\pi + 2\pi = \boxed{0.02\pi \approx 0.0628}$$