

Math 173 - Quiz 9

April 12, 2018

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3.5 points) Find and classify the critical points of
- $f(x, y) = y^4 - 4xy + x^2 + 7$
- .

$$f_x(x, y) = -4y + 2x = 0 \Rightarrow -4x = -8y$$

$$f_y(x, y) = 4y^3 - 4x = 0$$

$$4y^3 - 8y = 0$$

$$4y(y^2 - 2) = 0$$

$$y = 0, y = \sqrt{2}, y = -\sqrt{2}$$

$$x = 0, x = 2\sqrt{2}, x = -2\sqrt{2}$$

$$D(x, y) = \begin{vmatrix} 2 & -4 \\ -4 & 12y^2 \end{vmatrix} = 24y^2 - 16$$

$$D(0, 0) = -16 \Rightarrow (0, 0, 7) \text{ IS A SADDLE PT.}$$

$$D(2\sqrt{2}, \sqrt{2}) = 32 \text{ AND } f_{xx}(2\sqrt{2}, \sqrt{2}) = 2 > 0$$

$$\Rightarrow f(2\sqrt{2}, \sqrt{2}) = 3 \text{ IS A REL MIN.}$$

$$D(-2\sqrt{2}, -\sqrt{2}) = 32 \text{ AND } f_{xx}(-2\sqrt{2}, -\sqrt{2}) = 2 > 0$$

$$\Rightarrow f(-2\sqrt{2}, -\sqrt{2}) = 3 \text{ IS A REL MIN.}$$

2. (3.5 points) Use Lagrange multipliers to find the extreme values of $f(x, y) = xy$ subject to $3x^2 + y^2 = 6$.

$$g(x, y) = 3x^2 + y^2$$

$$\vec{\nabla} f(x, y) = y\hat{i} + x\hat{j}$$

$$\vec{\nabla} g(x, y) = 6x\hat{i} + 2y\hat{j}$$

$$y = \lambda 6x$$

$$x = \lambda 2y$$

$$3x^2 + y^2 = 6$$

$$y = \lambda^2 12y \Rightarrow$$

$$y = 0 \text{ or } \lambda = \pm \frac{1}{\sqrt{12}}$$

$$\Downarrow$$

BUT (0,0) IS IMPOSSIBLE.

$$\Downarrow y = \pm \frac{6}{\sqrt{12}}x$$

$$\Downarrow$$

$$3x^2 + \frac{36}{12}x^2 = 6$$

$$\Downarrow x = \pm 1$$

$$\begin{matrix} x=1 \\ y = \pm\sqrt{3} \end{matrix}$$

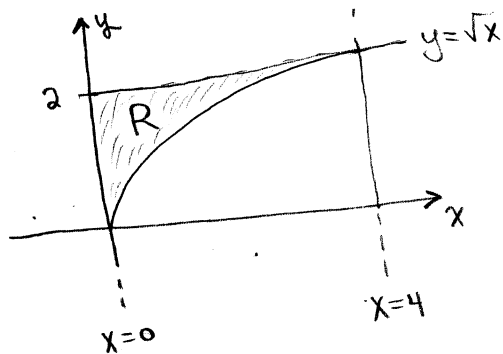
$$\begin{matrix} x=-1 \\ y = \pm\sqrt{3} \end{matrix}$$

$$f(\pm 1, \pm\sqrt{3}) = \sqrt{3} \quad \text{MAX}$$

$$f(\pm 1, \mp\sqrt{3}) = -\sqrt{3} \quad \text{MIN}$$

3. (3 points) Evaluate the iterated integral by reversing the order of integration.

$$\int_0^4 \int_{\sqrt{x}}^2 \sin y^3 dy dx$$



$$\int_{y=0}^2 \int_{x=0}^{y^2} \sin y^3 dx dy$$

$$= \int_0^2 y^2 \sin y^3 dy$$

$$u = y^3 \\ du = 3y^2 dy \quad \frac{1}{3} du = y^2 dy$$

$$\frac{1}{3} \int_{u=0}^{u=8} \sin u du = -\frac{1}{3} \cos u \Big|_0^8$$

$$= \frac{1}{3} - \frac{1}{3} \cos 8 \approx 0.3818$$