

Math 173 - Quiz 10

April 18, 2019

Name _____

Score _____

Show each step to receive full credit. Supply explanations when necessary.

1. (4 points) Use Lagrange multipliers to find the extreme values of $f(x, y) = y^2 - 4x$ subject to $x^2 + y^2 = 9$.

$$\vec{\nabla} f(x, y) = -4\hat{i} + 2y\hat{j}$$

$$\vec{\nabla} g(x, y) = 2x\hat{i} + 2y\hat{j}$$

$$-4 = 2\lambda x$$

$$2y = 2\lambda y \longrightarrow 2y - 2\lambda y = 0$$

$$\Rightarrow 2y(1-\lambda) = 0$$

$$y=0 \text{ or } \lambda=1$$

$$x^2 + y^2 = 9$$

$$x^2 = 9$$

$$x = \pm 3$$

$$x = -2$$

$$y^2 = 5$$

$$y = \pm\sqrt{5}$$

Four CRITICAL POINTS:

$$(3, 0), (-3, 0), (-2, \sqrt{5}), (-2, -\sqrt{5})$$

$$f(3, 0) = -12 \quad \} \quad \text{MIN VALUE IS } -12 \text{ AT } (3, 0)$$

$$f(-3, 0) = 12$$

$$f(-2, \sqrt{5}) = 13$$

$$f(-2, -\sqrt{5}) = 13$$

MAX VALUE IS 13 AT $(-2, \sqrt{5})$ AND $(-2, -\sqrt{5})$

2. (6 points) Consider the iterated integral $\int_0^4 \int_y^{2y} (8x + e^y) dx dy$.

(a) Evaluate the iterated integral. You'll need integration by parts. Show your work!

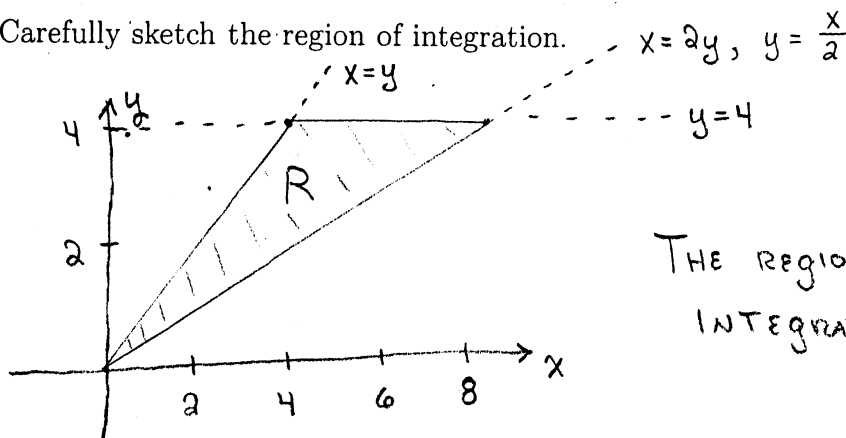
$$\int_0^4 (4x^2 + xe^y) \Big|_y^{2y} dy = \int_0^4 (12y^2 + ye^y) dy = 4y^3 \Big|_0^4 + ye^y \Big|_0^4 - \int_0^4 e^y dy$$

$$u=y \quad du=dy$$

$$dv=e^y dy \quad v=e^y$$

$$= 4y^3 \Big|_0^4 + ye^y \Big|_0^4 - e^y \Big|_0^4 = 256 + 3e^4 + 1 = \boxed{257 + 3e^4}$$

(b) Carefully sketch the region of integration.



THE REGION OF
INTEGRATION IS R.

(c) Set up the iterated integrals with the reversed order of integration. You'll need two iterated integrals.

$$\int_{x=0}^{x=4} \int_{y=x/2}^{y=x} (8x + e^y) dy dx + \int_{x=4}^{x=8} \int_{y=x/2}^{y=4} (8x + e^y) dy dx$$

$$e^4 - 2e^2 + \frac{259}{3}$$

$$+ 2e^4 + 2e^2 + \frac{512}{3}$$

$$= 3e^4 + 257$$