Math 173 - Quiz 5
February 28, 2019

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) An object is launched from the ground with an initial speed of 55 m/s at an initial angle of 25°. When does the object land on the ground? What is the horizontal distance traveled? (Use $g = 9.8\,\text{m/s}^2$ and ignore all forces except gravity.)

$$\vec{r}(t) = 55\cos 25°\, t\,\hat{i} + (-4.9t^2 + 55\sin 25°\, t)\,\hat{j}$$

$$-4.9t^2 + 55\sin 25°\, t = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{55\sin 25°}{4.9} \approx \boxed{4.7437 \text{ sec}}$$

$$\frac{55\cos 25° \cdot 55\sin 25°}{4.9} \approx \boxed{236.46 \text{ m}}$$

2. (2 points) Let $\vec{r}(t) = 4t\,\hat{i} + \sin 3t\,\hat{j} - \cos 3t\,\hat{k}$. Compute the unit tangent vector.

$$\vec{r}\,'(t) = 4\hat{i} + 3\cos 3t\,\hat{j} + 3\sin 3t\,\hat{k}$$

$$\|\vec{r}\,'(t)\| = \sqrt{16 + 9\cos^2 3t + 9\sin^2 3t} = \sqrt{25} = 5$$

$$\boxed{\hat{T}(t) = \frac{4}{5}\hat{i} + \frac{3}{5}\cos 3t\,\hat{j} + \frac{3}{5}\sin 3t\,\hat{k}}$$

TAKE-HOME PORTION OF QUIZ 5. DUE MONDAY.

3. (2 points) Compute the principal unit normal vector for the helix described by $\vec{r}(t) = a\cos t\,\hat{\imath} + a\sin t\,\hat{\jmath} + bt\,\hat{k}$, where a and b are positive real numbers.

$\vec{r}\,'(t) = -a\sin t\,\hat{\imath} + a\cos t\,\hat{\jmath} + b\,\hat{k}$

$\|\vec{r}\,'(t)\| = \sqrt{a^2+b^2}$

$\hat{T}(t) = \frac{1}{\sqrt{a^2+b^2}}\left(-a\sin t\,\hat{\imath} + a\cos t\,\hat{\jmath} + b\,\hat{k}\right)$

$\hat{T}\,'(t) = \frac{1}{\sqrt{a^2+b^2}}\left(-a\cos t\,\hat{\imath} - a\sin t\,\hat{\jmath}\right)$

$\hat{N}(t)$ = UNIT VECTOR IN DIRECTION OF $\hat{T}\,'(t)$

$= -\cos t\,\hat{\imath} - \sin t\,\hat{\jmath}$

4. (2 points) The graph of $\vec{r}(t) = \cos^3 t\,\hat{\imath} + \sin^3 t\,\hat{\jmath}$ for $0 \le t \le 2\pi$ is called a hypocycloid. Find the length of the hypocycloid.

$\vec{r}\,'(t) = 3\cos^2 t(-\sin t)\,\hat{\imath} + 3\sin^2 t(\cos t)\,\hat{\jmath}$

$\|\vec{r}\,'(t)\| = \sqrt{9\cos^4 t\sin^2 t + 9\sin^4 t\cos^2 t}$

$= \sqrt{9\cos^2 t\sin^2 t}\sqrt{\cos^2 t + \sin^2 t}$

$= 3|\cos t\sin t|$

$\int_0^{2\pi} 3|\cos t\sin t| =$

$4\int_0^{\pi/2} 3\cos t\sin t\,dt$ (By symmetry)

$= 12\int_0^1 u\,dt = 12\left(\frac{u^2}{2}\right)\Big|_0^1$ $\quad u = \sin t$

$= 6$

5. (1 point) Let a be a positive real number and let $\vec{r}(t) = t\,\hat{\imath} + at^2\,\hat{\jmath}$. The graph of $\vec{r}$ is a parabola. Find the curvature function.

$\vec{r}\,'(t) = \hat{\imath} + 2at\,\hat{\jmath}$ $\quad \|\vec{r}\,'(t)\| = \sqrt{1+4a^2t^2}$

$\vec{r}\,''(t) = 2a\,\hat{\jmath}$

$\vec{v}(t)\times\vec{a}(t) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2at & 0 \\ 0 & 2a & 0 \end{vmatrix}$

$= 2a\,\hat{k}$

$\kappa(t) = \frac{\|\vec{v}(t)\times\vec{a}(t)\|}{\|\vec{v}(t)\|^3}$

$\kappa(t) = \frac{2a}{(1+4a^2t^2)^{3/2}}$