Math 173 - Quiz 6

March 7, 2019

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Each of these equations describes a surface in 3D space. Identify the type of surface.

   (a) $x^2 - z^2 = y$ Hyperbolic paraboloid

   (b) $2x^2 + y^2 - z^2 = 1$ Hyperboloid of one sheet

   (c) $y^2 = 4x$ Parabolic cylinder

   (d) $2z = 4x^2 + 6y^2$ Paraboloid

   (e) $4x^2 + 8z^2 = y^2$ Cone

   (f) $x^2 + 3z^2 + 4 = y^2$ Hyperboloid of two sheets

2. (1.5 points) Consider the function $h(x, y) = \sqrt{1 + y - x^2}$.

   (a) What is the domain of h?

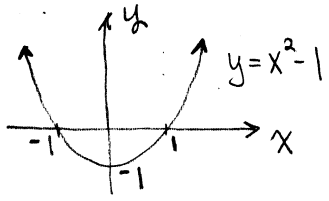
   $1 + y - x^2 \geq 0 \Rightarrow y \geq x^2 - 1$

   Domain $= \{(x, y) : y \geq x^2 - 1\}$

   All points above and on the parabola $y = x^2 - 1$

   (b) Sketch the level curve $h(x, y) = 0$.

   $h(x, y) = 0 \Rightarrow y = x^2 - 1$

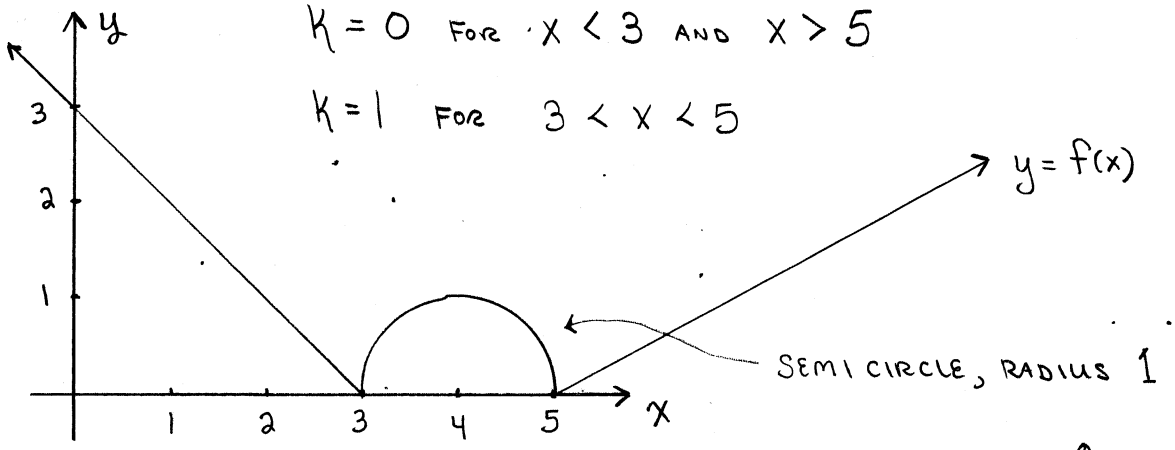
   (c) The graph of h is one-half of one of the quadric surfaces that we are familiar with. Describe the graph of h.

   Upper half of $z^2 = 1 + y - x^2$

   $y = x^2 + z^2 - 1$

   $\Rightarrow$ Upper half of a paraboloid.

3. (1.5 points) Sketch a curve that has at least one point at which the curvature is 1 and at least one point at which the curvature is 0. Identify and label such points.



$k = 0$ FOR $x < 3$ AND $x > 5$

$k = 1$ FOR $3 < x < 5$

4. (2 points) Find the limit or show that it does not exist: $\lim_{(x,y)\to(1,4)} \dfrac{xy - 2x - y + 2}{x^2y + x^2 - y - 1}$ $\frac{0}{0}$ MORE WORK

$$\lim_{(x,y)\to(1,4)} \frac{x(y-2) - 1(y-2)}{x^2(y+1) - 1(y+1)} = \lim_{(x,y)\to(1,4)} \frac{(x-1)(y-2)}{(x^2-1)(y+1)}$$

$$= \lim_{(x,y)\to(1,4)} \frac{y-2}{(x+1)(y+1)} = \frac{4-2}{(2)(5)} = \frac{2}{10} = \boxed{\frac{1}{5}}$$

5. (2 points) Find the limit or show that it does not exist: $\lim_{(x,y)\to(0,0)} \dfrac{2x - y^2}{2x^2 + y}$

Along $x = 0$: $\lim_{y\to 0} \frac{-y^2}{y} = \lim_{y\to 0} (-y) = 0$

Along $y = x$: $\lim_{x\to 0} \frac{2x - x^2}{2x^2 + x} = \lim_{x\to 0} \frac{2-x}{2x+1} = 2$

LIMIT DNE