Math 173 - Quiz 8

April 4, 2019

Name: key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the directional derivative of $f(x,y,z) = xye^z$ at $P(2,4,0)$ in the direction from P to $Q(0,0,0)$.

$\vec{PQ} = -2\hat{i} - 4\hat{j}$

$\|\vec{PQ}\| = \sqrt{4+16} = \sqrt{20}$

$\vec{\nabla} f(x,y,z) = ye^z\hat{i} + xe^z\hat{j} + xye^z\hat{k}$

$\vec{\nabla} f(2,4,0) = 4\hat{i} + 2\hat{j} + 8\hat{k}$

$D_{\vec{PQ}} f(2,4,0) = \frac{1}{\sqrt{20}}(-8-8) = \frac{-16}{\sqrt{20}}$

2. (3 points) Let $G(x,y) = \dfrac{8y}{1+x^2+y^2}$. Find a unit vector that points in the direction of maximum increase of G at the point $(2,1)$.

$$\vec{\nabla} G(x,y) = \frac{-16xy}{(1+x^2+y^2)^2}\hat{i} + \frac{(1+x^2+y^2)(8) - (8y)(2y)}{(1+x^2+y^2)^2}\hat{j}$$

$$\vec{\nabla} G(2,1) = \underbrace{\frac{-32}{36}\hat{i} + \frac{32}{36}\hat{j}}_{\text{HAS DIRECTION OF } -\hat{i}+\hat{j}}$$

$$\frac{\vec{\nabla} G(2,1)}{\|\vec{\nabla} G(2,1)\|} = \frac{1}{\sqrt{2}}(-\hat{i}+\hat{j})$$

3. (4 points) Find an equation of the plane that is tangent to the graph of $xy^2+3x = 8+z^2$ at the point $(1,-3,2)$.

$F(x,y,z) = xy^2 + 3x - z^2$

OUR SURFACE IS THE LEVEL SURFACE $F(x,y,z) = 8$.

$\vec{\nabla} F(x,y,z) = (3+y^2)\hat{i} + (2xy)\hat{j} - 2z\hat{k}$

$\vec{n} = \vec{\nabla} F(1,-3,2) = 12\hat{i} - 6\hat{j} - 4\hat{k}$

TAN PLANE ...

$12(x-1) - 6(y+3) - 4(z-2) = 0$

- OR -

$12x - 6y - 4z = 22$

- OR -

$6x - 3y - 2z = 11$