Math 200 - Test 1

September 19, 2012

Name key

Score _____

SHOW ALL WORK TO RECEIVE FULL CREDIT. SUPPLY EXPLANATIONS WHERE NECESSARY.

1. (2 points) Clearly state the four steps of the problem-solving process (in order).

① UNDERSTAND THE PROBLEM

② DEVISE A PLAN

③ CARRY OUT THE PLAN

④ LOOK BACK

2. (3 points) Which step of the problem-solving process is being used in each of these situations?

   (a) While solving a complicated equation, Annie double checked her work at each step.

   ③ CARRY OUT THE PLAN

   (b) After completely solving a problem, John solved the problem again by using an entirely different approach.

   ④ LOOK BACK

   (c) Stacy wrote, "Let x be the number of childrens' tickets."

   ① UNDERSTAND THE PROBLEM

   (d) Fred thought to himself, "My solution cannot be correct because the length of a rectangle cannot be -12 in."

   ④ LOOK BACK

   (e) A problem involved finding the perimeter of a rectangle. Sally decided to first find the length of the rectangle.

   ② DEVISE A PLAN

   (f) In order to solve a problem involving a trapezoid, Sam had to look up the definition of *trapezoid*.

   ① UNDERSTAND THE PROBLEM

3. (3 points) Sabrina bought two books at the book fair. One book cost \$3 more than the other, and because she bought two books, she got a \$2 discount. If she spent a total of \$12, how much was each book?

Let x be the cost of the cheaper book.

The more expensive book then cost $x+3$.

Book 1 + Book 2 − Discount = 12

$x + x + 3 - 2 = 12$

$2x + 1 = 12$

$2x = 11$

$x = 5.50$

The cheaper book was \$5.50.
The more expensive book was \$8.50.

Check: One book is \$3 more than the other.
And $5.50 + 8.50 - 2 = 12$.

4. (3 points) Tony builds regular bicycles and tandem (2-person) bicycles. Looking at his bicycles, he sees 18 wheels and 12 seats. How many of each type of bike does he have?

Two wheels per bike & 18 wheels $\Rightarrow$ 9 bikes

Let x = # of 1-seat bikes
y = # of 2-seat bikes

Bikes: $x + y = 9$

Seats: $x + 2y = 12$

$x + 2y = 12$
$-x - y = -9$
$y = 3 \Rightarrow x = 6$

6 regular bikes & 3 tandem bikes

Check: 6 regulars and 3 tandems make
$12 + 6 = 18$ wheels and
$6 + 6 = 12$ seats.

5. (4 points) Choose either one of the two problems above. For each step of the problem-solving process, list one strategy that you actually used in your solution. (Only list strategies that you actually used!)

Problem #3

Understand the problem: I used a variable to name my unknown and clearly defined the variable

Devise a plan: I wrote an equation involving my variable. My plan is to solve the equation.

Carry out the plan: I solved the equation.

Look back: I checked my solution in the original wording of the problem.

6. (1 point) Which one of the following is an example of inductive reasoning?

   (a) $2(3+5) = 2(5+3)$

   (b) A sequence begins with 2,4,6,8. The next term must be 10. ← GENERALIZATION BASED ON OBSERVATION

   (c) If $x = 10$, then $2x + 3 = 23$.

   (d) Wednesdays are pizza days, so today is a pizza day.

7. (1 point) Which one of these sets is well-defined?

   (a) The set of all tall women WHAT IS TALL?

   (b) $\{1, 2, 3, 4, 5\}$

   (c) $\{x \mid x \in \mathbb{N} \text{ and } x \text{ is big}\}$ WHAT IS BIG?

   (d) The set of all happy children WHAT IS HAPPY?

8. (1 point) Which one of these is a geometric sequence?

   (a) $1, 2, 3, 4, 5, \ldots$

   (b) $6, 7, 12, 21, 44, \ldots$

   (c) $1, 12, 123, 1234, \ldots$

   (d) $1, 2, 4, 8, 16, \ldots$ ← MULT BY 2

9. (1 point) Which one of the following is an example of deductive reasoning?

   (a) Every elephant I have ever seen is gray. Therefore every elephant must be gray.

   (b) It has rained every day. Therefore it will rain tomorrow.

   (c) Since the order of multiplication doesn't matter, $15 \cdot 37$ must be equal to $37 \cdot 15$. ← NOT BASED ON OBSERVATION

   (d) A sequence begins with 1,2,3,4. The next term must be 5.

10. (1 point) Find the 87th term of the following arithmetic sequence: $10, 13, 16, 19, \ldots$

    (a) 87

    (b) 268

    (c) 271

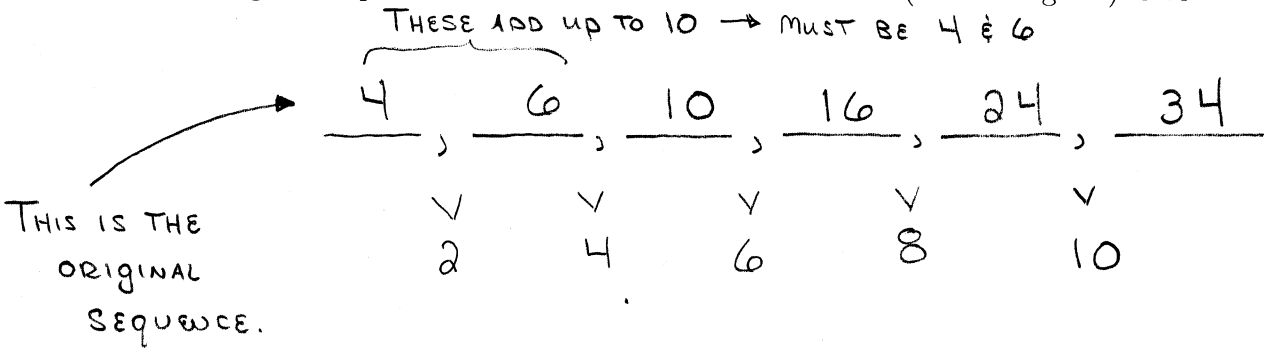
    (d) 254

N^{TH} TERM $= 3N + 7$

87^{TH} TERM $= 3(87) + 7$

$= 268$

11. (3 points) The 1st difference of a sequence is $2, 4, 6, 8, 10 \ldots$. Find the first six terms of the original sequence if the sum of the first two terms (of the original) is 10.



12. (3 points) A scientist notices that her bacteria culture doubles in population every hour. If she started an experiment with 100 bacteria, how many will she have after n hours? What type of sequence is being described in this problem?

100, 200, 400, 800, ...

GEOMETRIC SEQUENCE

N^{TH} TERM $= 100 \cdot 2^{N-1}$

13. (3 points) Translate into words and then rewrite in roster notation.

$$\{x \mid x \in \mathbb{N} \text{ and } x \leq 8.9\}$$

SET OF ALL X'S SUCH THAT X IS A NATURAL NUMBER AND X IS LESS THAN OR EQUAL TO 8.9

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

14. (2 points) Rewrite in set-builder notation: $\{5, 6, 7, 8, 9\}$

$\{5, 6, 7, 8, 9\} = \{x \mid x \in \mathbb{N} \text{ AND } 5 \leq x \leq 9\}$

15. (3 points) Find a formula for the nth term of each sequence.

(a) $1, 8, 15, 22, 29, \ldots$

7 7 7 7 ARITHMETIC w/ DIFF = 7

N^{TH} TERM $= 7N - 6$

(b) $17, 21, 25, 29, 33, \ldots$

4 4 4 4 ARITHMETIC w/ DIFF = 4

N^{TH} TERM $= 4N + 13$

(c) $2, 10, 50, 250, 1250, \ldots$

GEOMETRIC w/ RATIO = 5

N^{TH} TERM $= 2 \cdot 5^{N-1}$

16. (2 points) Find the next two terms: $6, 7, 12, 21, 34, \ldots$

1 5 9 13 17 21

4 4 4 4

$34 + 17 = 51$

$51 + 21 = 72$ } NEXT TWO TERMS

17. (2 points) Give a counterexample: For any number x, $(x+1)^2 = x^2 + 1$.

$X = 5$:

$(5+1)^2 = 6^2 = 36 \neq 5^2 + 1 = 26$

18. (2 points) Let X be the set of letters of the word *racecar*.

(a) Write X is roster notation.

$X = \{ r, a, c, e \}$

(b) Find $n(X)$.

$n(X) = 4$

19. (3 points) Suppose $A = \{1, 3, 7\}$, $B = \{x, y, z\}$ and $C = \{3, 6, 9, 12\}$. Indicate whether each statement is true or false.

(a) $n(A) = 7$ F

(b) $3 \in A$ T

(c) $A \sim C$ F

(d) B is well-defined. T

(e) $A = B$ F

(f) A and C are in 1–1 correspondence. F

20. (4 points) Consider the sum: $3 + 8 + 13 + \cdots + 2083$

(a) How many terms does the sum have?

N^{TH} TERM $= 5N - 2$ $\quad 5N - 2 = 2083$

$5N = 2085 \Rightarrow$ **$N = 417$**

(b) Compute the sum.

$3 + 8 + \cdots + 2083$

$2083 + \cdots + 3$

$2086 \quad 2086 \quad 2086$

Sum $= \dfrac{(2086)(417)}{2}$

$= 434,931$

21. (1 point) Kate asked George to write the elements of the set $\{5, 10, 15\}$. George wrote: $\{5\}$, $\{10\}$, $\{15\}$. Is George correct? Explain.

No, THE ELEMENTS OF THE SET ARE THE NUMBERS 5, 10, 15, NOT THE SETS $\{5\}$, $\{10\}$, $\{15\}$.

22. (2 points) Find the cardinality of the set $\{2, 5, 8, \ldots, 320\}$.

N^{TH} TERM $= 3N - 1$

$3N - 1 = 320$

$3N = 321 \Rightarrow$ **$N = 107$**

CARDINALITY IS 107.