Math 200 - Test 3

Name key

November 21, 2012

Score _____

Show all work. Supply explanations when necessary.

1. (1 point) Choose the multiplication model that best fits the following problem situation: *The graduates walked into the auditorium as a group in ten rows of four. How many graduates were there?*

   (a) area/array
   (b) repeated addition
   (c) set partition
   (d) Cartesian product

2. (1 point) Estimate the following sum by rounding each addend to the nearest hundred: $550 + 147 + 49 + 1449 + 696$

   (a) 2800
   (b) 2900
   (c) 2850
   (d) 3000

   $600 + 100 + 0 + 1400 + 700$

3. (1 point) What algorithm is being illustrated here?

$$\begin{array}{r} 62 \\ -\,38 \\ \hline \end{array} \longrightarrow \begin{array}{r} 62 \;+\; 2 \\ -\,(38 \;+\; 2) \\ \hline \end{array} \longrightarrow \begin{array}{r} 64 \\ -\,40 \\ \hline 24 \end{array}$$

   (a) nice-numbers algorithm
   (b) fast subtraction algorithm
   (c) scratch subtraction algorithm
   (d) equal-additions algorithm

4. (1 point) Suppose $A = 2^3 \cdot 5^2 \cdot 7 \cdot 13^3$ and $B = 2 \cdot 3 \cdot 5^3 \cdot 13^2$. Find the GCD of A and B.

   (a) $2 \cdot 3 \cdot 5^2 \cdot 7 \cdot 13^2$
   (b) $2^3 \cdot 3 \cdot 5^3 \cdot 7 \cdot 13^3$
   (c) $2 \cdot 5^2 \cdot 13^2$
   (d) $2^4 \cdot 3 \cdot 5^5 \cdot 7 \cdot 13^5$

   $GCD(A,B) = 2 \cdot 5^2 \cdot 13^2$

5. (5 points) Use any nontraditional algorithm to compute each product. Use a different algorithm for each part.

(a) 678×54

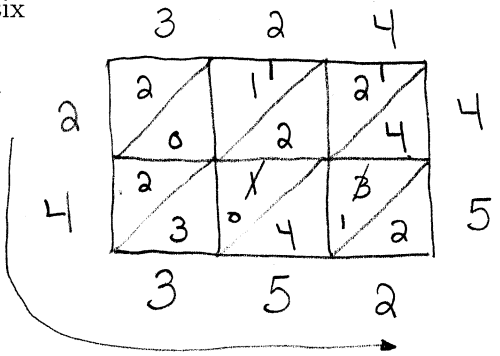
PARTIAL PRODUCTS...

$$\begin{array}{r} 678 \\ \times\ 54 \\ \hline 32 \\ 280 \\ 2400 \\ 400 \\ 3500 \\ 30000 \\ \hline 36612 \end{array}$$

36,612

(b) $45_{six} \times 324_{six}$

LATTICE...

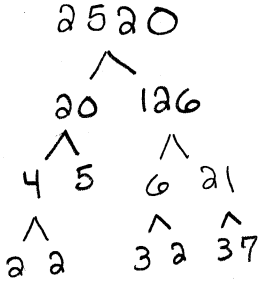


$24\,352_{six}$

6. (3 points) Write an application problem (i.e. a word problem) involving the division fact $21 \div 3 = 7$ where the division is best represented by the partition (sharing) model.

JOEY HAS 21 MARBLES THAT HE SPLITS UP EVENLY INTO 3 GROUPS. HOW MANY MARBLES WILL BE IN EACH GROUP?

7. (4 points) Find the prime factorization of 2520. Then determine how many whole number divisors it has.



OF DIVISORS = $4 \times 3 \times 2 \times 2 = 48$

$2520 = 2^3 \cdot 3^2 \cdot 5 \cdot 7$

8. (2 points) Compute the following sum: $143_{\text{five}} + 244_{\text{five}} + 313_{\text{five}} + 342_{\text{five}}$

(a) 2202_{five}

(b) 1042_{five}

(c) 1212_{five}

(d) 3212_{five}

$$\begin{array}{r} {\scriptstyle 3\ 2} \\ 143_{\text{five}} \\ 244_{\text{five}} \\ 313_{\text{five}} \\ 342_{\text{five}} \\ \hline 2202_{\text{five}} \end{array}$$

9. (1 point) Which one of the following is a true statement?

(a) 10 is divisible by 20

(b) $3 \mid 74052$

(c) $0 \mid 1$

(d) $6 \mid 3 = 2$

10. (1 point) Which one of the following numbers is NOT prime?

(a) 797

(b) 53

(c) 23

(d) 1

11. (1 point) What is the greatest prime number you must consider to test whether 1343 is prime?

(a) 79

(b) 31

(c) 36

(d) 23

$\sqrt{1343} \approx 36.65$

12. (1 point) Choose the division model that best fits the following problem situation: *Jenny has 35 pictures, and she intends to put 5 pictures on each page of her scrapbook. How many pages of the scrapbook will she use?*

(a) repeated subtraction

(b) missing factor

(c) partition

(d) Cartesian quotient

13. (4 points) Use any method (not your calculator) to compute the greatest common divisor of 2720 and 736.

$$\begin{array}{r} 3 \\ 736\overline{)2720} \\ -2208 \\ \hline 512 \end{array}$$

$$\begin{array}{r} 1 \\ 512\overline{)736} \\ -512 \\ \hline 224 \end{array}$$

$$\begin{array}{r} 2 \\ 224\overline{)512} \\ 448 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 3 \\ 64\overline{)224} \\ 192 \\ \hline 32 \end{array}$$

$\text{GCD}(2720, 736)$

$= \text{GCD}(736, 512)$

$= \text{GCD}(512, 224)$

$= \text{GCD}(224, 64)$

$= \text{GCD}(64, 32) = \boxed{32}$

14. (3 points) In each case, use a different strategy to estimate the sum.

(a) $453 + 397 + 405 + 367 + 398 + 421 + 347 \approx 7 \times 400$

Clustering ...

$= \boxed{2800}$

(b) $105 + 478 + 690 + 342 + 821 \approx 100 + 500 + 700 + 300 + 800$

Rounding to nearest hundred...

$= \boxed{2400}$

(c) $12 + 69 + 49 + 80 + 54 + 35 + 90$

Nice numbers...

$90 + 12 \approx 100$

$69 + 35 \approx 100$

$49 + 54 \approx 100$

$80 = 80$

$\boxed{380}$

15. (5 points) Test the following number for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, and 11. Give and explain your conclusion for each test.

3 9 2 1 4 4 9 4 3 3 0

By 2: YES BECAUSE 2 | 0

By 3: 3+9+2+1+4+4+9+4+3+3+0 = 42

YES BECAUSE 3 | 42

By 4: NO BECAUSE 4 ∤ 30

By 5: YES BECAUSE 5 | 0

By 6: YES BECAUSE DIVISIBLE BY 3 & 2

By 8: NO, NOT BY 4

By 9: NO, 9 ∤ 42

By 10: YES, BECAUSE 10 | 0

By 11: 3+2+4+9+3+0 = 21

9+1+4+4+3 = 21

21-21 = 0

YES BECAUSE 11 | 0

16. (3 points) Use the standard long division algorithm to compute the quotient and remainder when 57,354 is divided by 7. Then use the short division algorithm.

STANDARD

$$\begin{array}{r} 8193 \text{ r } 3 \\ 7\overline{)57,354} \\ -56 \\ \hline 13 \\ -7 \\ \hline 65 \\ -63 \\ \hline 24 \\ 21 \\ \hline 3 \end{array}$$

SHORT

$$7\overline{)57\,35\,4} \quad 8193 \text{ r } 3$$

17. (3 points) Suppose that $d \mid a$ and $d \mid b$. Which one of the following is NOT necessarily true? After choosing your answer, give a counterexample.

(a) $a \mid b$

(b) $d \mid (a+b)$

(c) $d \mid (a-b)$

(d) $d \mid ab$

3 | 6 AND 3 | 9

BUT

6 ∤ 9

18. (4 points) Use any nontraditional algorithm to compute each of the following.

(a) $4598 + 6894$

LATTICE...

$$\begin{array}{r} 4\ 5\ 9\ 8 \\ +\ 6\ 8\ 9\ 4 \end{array}$$

1	1	1	1
0	3	8	2

$1\ 1\ 4\ 9\ 2$

$\boxed{11,492}$

(b) $647 - 368$

EQUAL ADDITIONS...

$$\begin{array}{r} 647 + 2 \\ -\ 368 + 2 \\ \hline \end{array} \rightarrow \begin{array}{r} 649 + 30 \\ -\ 370 + 30 \\ \hline \end{array} \rightarrow \begin{array}{r} 679 \\ -\ 400 \\ \hline \end{array}$$

$\boxed{279}$

19. (3 points) Carefully explain why $8 \div 0$ is not defined.

$8 \div 0 = C$ IF AND ONLY IF C IS THE UNIQUE NUMBER SUCH THAT $C \times 0 = 8$.

THERE IS NO SUCH NUMBER C!

20. (3 points) Find the prime factorization of the smallest positive number divisible by all of 2, 3, 4, 5, 6, 7, 8, 9, and 10.

$2 \cdot 3 \cdot 2 \cdot 5 \cdot 7 \cdot 2 \cdot 3 = \boxed{2^3 \cdot 3^2 \cdot 5 \cdot 7}$

$= 2520$

21. (4 points extra credit) Consider the number whose expanded form is

$$9 \times 10^{25} + 2 \times 10^{24} + 5 \times 10^{13} + 7 \times 10^{9} + 8 \times 10^{2}.$$

Test this number for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, and 11.

SINCE THE NUMBER ENDS WITH ZERO, IT IS DIVISIBLE BY 2, 5, 10

LAST TWO DIGITS ARE 00. IT IS DIVISIBLE BY 4

LAST THREE DIGITS ARE 800. IT IS DIVISIBLE BY 8.

SUM OF DIGITS IS $9+2+5+7+8 = 31$. IT IS NOT DIVISIBLE BY 3 OR 9

SINCE IT IS NOT DIVISIBLE BY 3, IT IS NOT DIVISIBLE BY 6.

SUMS OF EVERY OTHER DIGIT

ARE $9+5+7 = 21$

AND $2+8 = 10$.

$21-10 = 11 \Rightarrow$ IT IS DIVISIBLE BY 11.

22. (1 point extra credit) The Russian peasant algorithm is a multiplication algorithm that is described in the exercise set of section 3.4. Use the Russian peasant algorithm to compute 97×38.

	Halves	Doubles
	38	97
★	19	194
★	9	388
	4	776
	2	1552
★	1	3104

$3104 + 388 + 194 = 3686$

$97 \times 38 = 3686$