

Math 200 - Test 1
February 16, 2011

Name key Score _____

Show all work to receive full credit (even on multiple-choice problems). Supply explanations when necessary.

1. (4 points) Clearly state the four steps of the problem-solving process (in order).

- ① UNDERSTAND THE PROBLEM
- ② DEVISE A PLAN
- ③ CARRY OUT THE PLAN
- ④ LOOK BACK

2. (3 points) State three different strategies for devising a plan.

- MAKE A TABLE OR DIAGRAM
- LOOK AT A SIMILAR PROBLEM
- WRITE AN EQUATION
- WORK BACKWARDS
- GUESS AND CHECK
- LOOK FOR A PATTERN, etc.

3. (3 points) Use the technique we discussed in class to compute the sum:

$$\begin{array}{r} 1 + 2 + 3 + 4 + \dots + 247 + 248. \\ 248 + 247 + \dots + 2 + 1 \\ \hline 248 \text{ PAIRS OF } 249 \end{array}$$

$$\text{Sum is } \frac{248(249)}{2} = \boxed{30,876}$$

4. (1 point) When using the 4-step, problem-solving process which one of these strategies WOULD NOT BE considered part of carrying out the plan?

- (a) Implement the plan you devised.
- (b) Keep a record of your work.
- (c) Check each step.
- (d) Try other possible solution methods. ← PART OF LOOK BACK

5. (1 point) What type of reasoning are you using when you draw a conclusion based on observation?

- (a) Inductive reasoning
- (b) Recursive thinking
- (c) Deductive reasoning
- (d) Circular reasoning

6. (1 point) What is the name of the type of sequence shown here?

8, 17, 26, 35, 44, 53, ...

✓ ✓ ✓ ✓ ✓
9 9 9 9 9

← FIXED DIFFERENCE

- (a) Geometric sequence
- (b) Arithmetic sequence
- (c) Sequence with fixed higher-order difference
- (d) Random sequence

7. (1 point) Which one of these is not a well-defined set?

- (a) {3, 4, 11, 18, 22, 25, 32, 36, 43}
- (b) \emptyset
- (c) The set of all freshwater fish
- (d) The set of all great books

WHAT IS A "GREAT" BOOK?

8. (1 point) Which one of the following sets can be placed into a one-to-one correspondence with the set of days of the week?

- (a) \emptyset
- (b) The set of real numbers less than 7
- (c) {1, x, 3, F, 13, l, g} ← 7 ELEMENTS
- (d) {s, m, t, w, f}

9. (1 point) Find the 915th term of the following arithmetic sequence:

1, 5, 9, 13, 17, 21, 25, ...

✓ ✓ ✓ ✓ ✓ ✓
4 4 4 4 4 4

- (a) 3657
- (b) 3660
- (c) 3661
- (d) 2915

NTH TERM IS $4N - 3$

$$4(915) - 3 =$$

10. (3 points) Is the following statement true or false? After stating your answer, give two examples or two counterexamples.

$$\text{For any whole number } x, (x+5)^2 = x^2 + 25.$$

FALSE. COUNTEREXAMPLES:

$$\textcircled{1} \quad x = 1 : (1+5)^2 = 6^2 = 36 \neq 1^2 + 25 = 26$$

$$\textcircled{2} \quad x = 10 : (10+5)^2 = 15^2 = 225 \neq 10^2 + 25 = 125$$

11. (2 points) When referring to sets, what is the difference between *equal* and *equivalent*?

SETS ARE EQUAL IF THEY HAVE EXACTLY THE SAME ELEMENTS.

SETS ARE EQUIVALENT IF THEY CAN BE PUT INTO A ONE-TO-ONE CORRESPONDENCE.

12. (1 point) Let Z be the set of the letters of the word *excellent*. Write Z in roster (listing) notation.

$$Z = \{ e, x, c, l, n, t \}$$

13. (1 point) For two sets A and B , what must be true if $A \not\subseteq B$?

THERE MUST BE AN ELEMENT OF A
THAT IS NOT AN ELEMENT OF B .

14. (2 points) Is 9500 a term of the following sequence?

$$8, 15, 22, 29, 36, 43, \dots$$

$$\begin{array}{cccccc} \vee & \vee & \vee & \vee & \vee \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 7 & 7 & 7 & 7 & 7 \end{array}$$

$$N^{\text{TH}} \text{ TERM} = 7N + 1$$

$$7N + 1 = 9500$$

$$7N = 9499$$

$$N = 1357$$

YES, 9500 IS THE
1357TH TERM.

15. (1 point) What does it mean for a set to be well defined?

THERE MUST BE A CLEAR WAY OF DETERMINING
WHAT IS OR IS NOT IN THE SET.

16. (3 points) On day 1, a woman discovers a great new shampoo. On day 2, she tells two friends. On day 3, each of her friends tells two friends. This pattern continues.

(a) Make a table showing the day number and the number of people learning about the shampoo on that day. Include the first 5 days in your table.

Day	# OF PEOPLE FINDING OUT THAT DAY
1	1
2	2
3	4
4	8
5	16

(b) What kind of sequence is described by your table? Find a formula for the n th term of the sequence.

GEOMETRIC SEQUENCE,

$$N^{\text{TH}} \text{ TERM} = 2^{N-1}$$

17. (1 point) Which of one of these mathematicians is associated with the following famous sequence?

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

(a) Bernhard Riemann

(b) Gerolamo Cardano

(c) Fibonacci

(d) Carl Friedrich Gauss

↑
FIBONACCI SEQUENCE

18. (4 points) A farmer needs to fence a rectangular field. She wants the length of the field to be 80 ft longer than the width. If she intends to use 1080 ft of fencing material, what should the length and width of the field be?

ALGEBRAIC APPROACH:

$$2w + 2l = 1080$$

$$2w + 2(w + 80) = 1080$$

$$2w + 2w + 160 = 1080$$

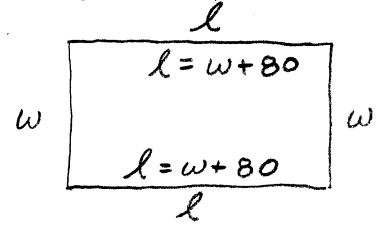
$$4w = 920$$

$$w = 230 \text{ FT}$$

$$l = 310 \text{ FT}$$

GUESS & CHECK:

WIDTH	LENGTH	PERIMETER
100	180	560
200	280	960
250	330	1160
225	305	1060
230	310	1080



$l = \text{LENGTH}$
 $w = \text{WIDTH}$

▶ 230 FT x 310 FT

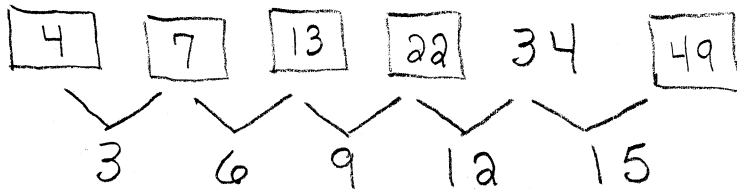
19. (1 point) Write the following set in roster (listing) notation.

$$\{x \mid x = 3n, \text{ and } n \in \mathbb{N}\}$$

↑ ↑
MULTIPLES N WILL TAKE THE VALUES
OF 3. 1, 2, 3, 4, ...

$$\{3, 6, 9, 12, 15, 18, \dots\}$$

20. (3 points) The first difference of a sequence is 3, 6, 9, 12, 15, ... The fifth term of the original sequence is 34. Find the first six terms of the original sequence.



ORIGINAL IS 4, 7, 13, 22, 34, 49, ...

21. (2 points) List all the subsets of $\{a, b\}$.

THERE ARE 4 OF THEM:

$$\emptyset, \{a\}, \{b\}, \{a, b\}$$

22. (3 points) Rewrite each of the following statements using mathematical symbols.

(a) b is an element of the set M .

$$b \in M$$

(b) The empty set is a subset of the set B .

$$\emptyset \subseteq B$$

(c) The cardinality of the set Z is 24.

$$n(Z) = 24$$

23. (3 points) Let $A = \{2, 5, 8, 11, 14, \dots, 650\}$. Find $n(A)$.

ELEMENTS OF A ARE TERMS OF AN

ARITHMETIC SEQUENCE: N TH TERM = $3N - 1$

$$3N - 1 = 650 \Rightarrow 3N = 651 \Rightarrow N = 217$$

$$n(A) = 217$$

24. (1 point) Let U be the set of all PSC students, and let M be the set of all PSC math students. Describe an element of the set \overline{M} .

AN ELEMENT OF \overline{M} IS A PSC STUDENT

WHO IS NOT A MATH STUDENT.

25. (3 points) Let $A = \{2, 4, 6, 8\}$, $B = \{2, 6, 8\}$, and $C = \{2, 3, 4\}$. In each of the following, insert an appropriate symbol \sim , \in , or \subseteq to make the statement true.

(a) $B \subseteq A$

(b) $\{3\} \subseteq C$

(c) $C \sim B$