Math 200 - Test 3

April 20, 2011

Name key

Score _____

Show all work. Supply explanations where necessary.

1. (1 point) Which one of the following is the additive inverse of the expression $3-4x+7y$?

   (a) 1

   (b) $-3+4x-7y$

   (c) $3+4x-7y$

   (d) $\dfrac{1}{3-4x+7y}$

   OPPOSITE IS $-3+4x-7y$

2. (1 point) Suppose x is negative integer. Which one of the following is true of $-x$?

   (a) $-x$ is less than zero

   (b) $-x$ is a negative integer

   (c) $-x$ is a positive integer

   (d) $-x$ may be either positive or negative

   OPPOSITE OF A NEGATIVE IS A POSITIVE.

3. (1 point) Choose the division model that best fits the following problem situation: *Sabrina can send a total of 250 text messages. If she sends 25 each day, for how many days can she send messages?*

   (a) missing factor model

   (b) charged field model

   (c) set partition model

   (d) repeated subtraction model

   HOW MANY GROUPS?

4. (1 point) Suppose p and q are integers. Which one of the following is equal to $-p \times (-q)$?

   (a) $p \times q$

   (b) $-(p \times q)$

   (c) $-p \times q$

   (d) $p \times (-q)$

5. (1 point) Consider the following conjecture:

   If $x \mid (y+7)$, then $x \mid y$.

   Which one of the following is a counterexample?

   (a) $7 \mid (14+7)$ and $7 \mid 14$

   (b) $5 \nmid (10+7)$ and $5 \mid 10$

   (c) $3 \mid (5+7)$ and $3 \nmid 5$.

   (d) The conjecture is true.

6. (3 points) Use a multiplication model to illustrate the following fact.

$$3 \cdot (2 + y) = 3 \cdot 2 + 3 \cdot y$$

AREA MODEL IS ALSO GOOD!

REPEATED ADDITION...

$$3 \cdot (2+y) = (2+y) + (2+y) + (2+y)$$
$$= (2+2+2) + (y+y+y) = 3 \cdot 2 + 3 \cdot y$$

7. (3 points) Suppose n is a positive number. Give a brief but thorough explanation for why $n \div 0$ is not defined.

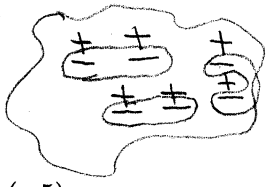
MISSING FACTOR MODEL: $N \div 0 = k$ IF k IS THE UNIQUE NUMBER SUCH THAT $k \cdot 0 = N$.

SINCE $N \neq 0$, THERE IS NO SUCH k.

8. (6 points) Use a different model to illustrate each product.

(a) $-3 \times (-2)$ CHARGES...

START WITH ZERO AND TAKE OUT 3 GROUPS OF 2 -'s. LEFT WITH +6.



(TAKE OUT CIRCLED GROUPS) $-3 \times (-2) = 6$

(b) $4 \times (-5)$

REPEATED ADDITION...

$$4 \times (-5) = (-5) + (-5) + (-5) + (-5) = -20$$

(c) -2×8

PATTERNS...

DECREASE FACTOR BY 1 ↓

$3 \times 8 = 24$ ↓ DECREASE PRODUCT BY 8

$2 \times 8 = 16$

$1 \times 8 = 8$

$0 \times 8 = 0$

$-1 \times 8 = -8$

$-2 \times 8 = -16$

9. (1 point) Which one of the following is a valid test for divisibility by 8?

   (a) An integer is divisible by 8 if and only if the sum of its digits is divisible by 8.

   (b) An integer is divisible by 8 if and only if its ones digit is divisible by 8.

   (c) An integer is divisible by 8 if and only if its last digit is an 8.

   (d) An integer is divisible by 8 if and only if the integer formed by the last three digits is divisible by 8.

10. (1 point) Choose the division model that best fits the following problem situation: *Tara has 24 stickers to share evenly among her 4 friends. How many stickers does each friend get?*

    (a) set partition model

    (b) charged field model

    (c) repeated subtraction model

    (d) missing factor model

How MANY IN EACH group?

11. (1 point) Which one of the following means the same as $a \mid b$?

    (a) a divided by b

    (b) a is a multiple of b

    (c) a is a divisor of b

    (d) There exists an integer k such that $a \div b = k$.

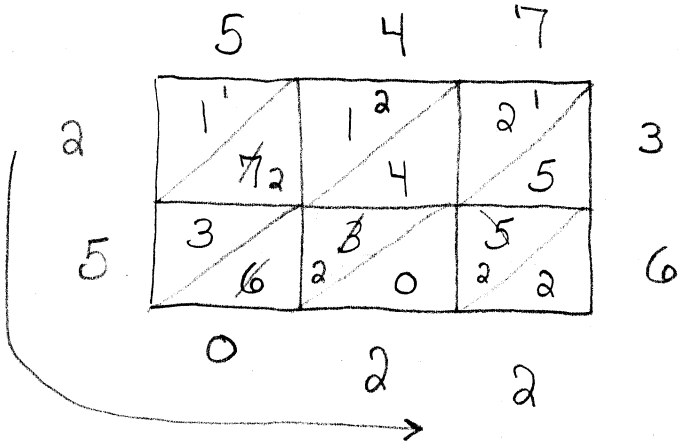
12. (3 points) Use any multiplication algorithm to compute $547_{\text{eight}} \times 36_{\text{eight}}$.

    (a) $25\,022_{\text{eight}}$

    (b) $19\,692_{\text{eight}}$

    (c) $24\,102_{\text{eight}}$

    (d) $22\,602_{\text{eight}}$



$25\,022_{\text{EIGHT}}$

13. (3 points) Zasha used the number line to model $-5-2$. Here is what she said:

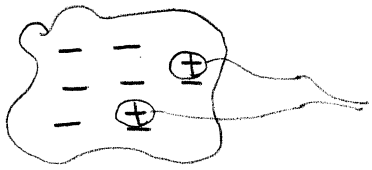
"Start at zero facing right. Turn around, go five. Back-up two. You end up at negative three. Therefore, $-5-2=-3$."

(a) Is Zasha correct? If not, correct her application of the number line model.

No, $-5-2=-7$. "Start at zero facing right. Back up 5. Turn around (to face left). Go foward 2. End at -7."

(b) Use a different model to illustrate Zasha's problem.

① Start with -5. ② Put in 2 neutral $\pm$ pairs. ③ Take out 2 +'s.



Take out. Leaves 7 -'s.

(c) What fact would Zasha be modeling if she said,

"Start at zero facing right. Back-up five. Then back-up two. You end up at negative seven."

$$-5+(-2)=-7$$

14. (3 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

Subtract their absolute values, least from greatest. Give the result the sign of the addend with the greatest absolute value.

Ex

$$-5+3=-(5-3)=-2$$

15. (1 point) Suppose x and y are integers.
Which one of the following is equal to $-x - (-y)$?

(a) $-x + y$
(b) $x - y$
(c) $-x - y$
(d) $x + y$

16. (1 point) The number 1 is also know as

(a) the multiplicative identity.
(b) the additive identity.
(c) the multiplicative inverse.
(d) the additive inverse.

17. (1 point) Which one of the following is a true statement?

(a) $0 \mid 13$
(b) $3 \mid 6321$
(c) $32 \mid 8 = 4$
(d) 25 divides 5

18. (1 point) Suppose $d \mid a$ and $d \mid b$. Which one of the following is NOT necessarily true?

(a) $d \mid (5a + 6b)$
(b) $a \mid d$
(c) $d \mid 7ab$
(d) $a \mid ad$ if $a \neq 0$

19. (1 point) Suppose x is an integer. Which one of the following is equal to $-x \div (-2)$?

(a) $-x \div 2$
(b) $-(x \div 2)$
(c) $x \div (-2)$
(d) $x \div 2$

20. (3 points) Explain why the algorithm illustrated below works.
Then use it to compute 37×56.

$$\begin{array}{r} 37 \\ \times\ 56 \\ \hline 42 \\ 180 \\ 350 \\ 1500 \\ \hline 2072 \end{array}$$

$$\begin{array}{r} 14 \\ \times\ 23 \\ \hline 12 \\ 30 \\ 80 \\ 200 \\ \hline 322 \end{array}$$

← THIS IS A PARTIAL PRODUCTS ALGORITHM. HERE ARE TWO WAYS TO SEE IT:

①

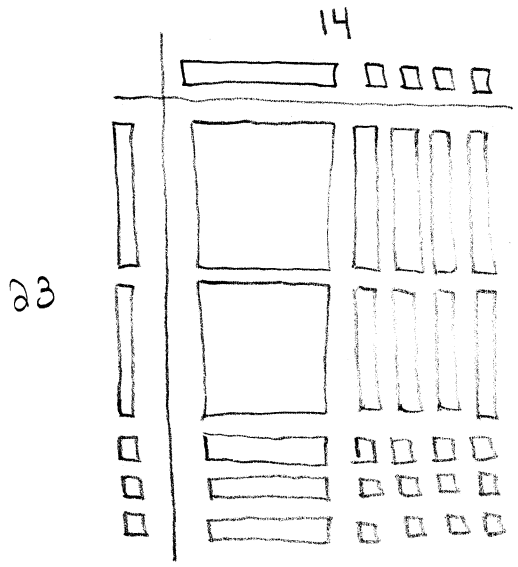
	10	4
20	200	80
3	30	12

↑ 322

② $10 + 4$
$\times\ 20 + 3$

$$\begin{array}{r} 12 \\ 30 \\ 80 \\ 200 \\ \hline 322 \end{array}$$

21. (3 points) Use base-10 blocks to illustrate the multiplication fact $14 \times 23 = 322$.



2 FLATS, 11 LONGS, 12 UNITS
↓
2 FLATS, 12 LONGS, 2 UNITS
↓
3 FLATS, 2 LONGS, 2 UNITS
↓
322

22. (2 points) State the test for divisibility by 9 and illustrate your test with a 4-digit example.

AN INTEGER IS DIVISIBLE BY 9 IF AND ONLY IF THE SUM OF ITS DIGITS IS DIVISIBLE BY 9.

EX 6345 IS DIVISIBLE BY 9

SINCE $6+3+4+5 = 18$

AND 9 DIVIDES 18.

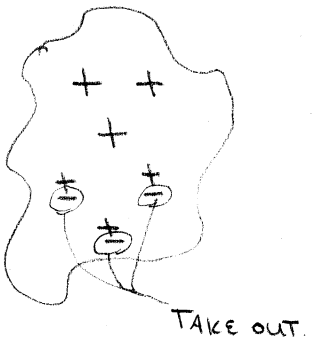
23. (3 points) It is common for students to make mistakes when computing differences such as $3-(-3)$ and $-3-(-3)$. Use the charged-field model to compute each difference. Be sure to label which is which.

$3-(-3)$

① START WITH 3 +'s AND 3 ± NEUTRAL PAIRS.

② TAKE OUT 3 -'s

③ LEFT WITH 6 +'s



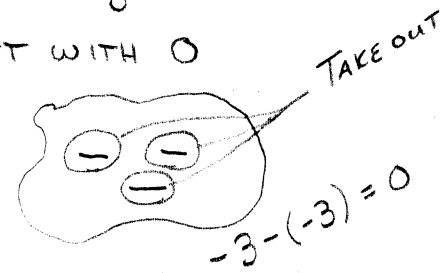
$3-(-3)=6$

$-3-(-3)$

① START WITH 3 -'s

② TAKE AWAY 3 -'s

③ LEFT WITH 0



$-3-(-3)=0$

24. (3 points) Test the following number for divisibility by (2), (3), (4), (5), 8, 9, and (10). Explain your reasoning.

8271509510940 → SUM OF DIGITS IS 51

By 2: YES, BECAUSE LAST DIGIT IS 0 AND $2 \mid 0$.

By 3: YES, BECAUSE SUM OF DIGITS IS 51 AND $3 \mid 51$.

By 4: YES, BECAUSE $4 \mid 40$

By 5: YES, BECAUSE LAST DIGIT IS 0.

By 8: NO, BECAUSE $8 \nmid 940$

By 9: NO, BECAUSE $9 \nmid 51$ (SUM OF DIGITS)

By 10: YES, BECAUSE LAST DIGIT IS ZERO.

$$8\overline{)9494}\quad 1184$$

25. (2 points) Use short division to compute $-34368 \div (-6)$. Make sure to mention how you deal with the signs.

SAME ANSWER AS $34368 \div 6$

$$6\overline{)34368}\quad 5728$$

$-34368 \div (-6) = \boxed{5728}$

26. (2 pts ex cred) See problem #8 on page 172. Use the Russian peasant algorithm to compute 98×73.

HALVES	DOUBLES
98	73
49	146
24	292
12	584
6	1168
3	2336
1	4672

4672
2336
146
7154

$98 \times 73 = 7154$