Math 200 - 1st Final Exam

May 4, 2011

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. Multiple choice problems are worth 0, 1, or 2 points depending on your answer and the work shown.

1. Choose the division model that best fits the following problem situation: *Sammy's mother made 36 cookies for the school bake sale. She would like to put them into bags that hold 4 cookies each. How many bags does she need?*

   (a) partition
   (b) missing factor
   (c) repeated subtraction
   (d) Cartesian product

   How many groups?

2. Which one of the following facts illustrates the commutative property of multiplication?

   (a) $(2+x)+y = y+(2+x)$
   (b) $3\cdot(x+7)+2 = 3x+23$
   (c) $3(2)(x+y) = 2(3)(x+y)$
   (d) $(2x+3y)+8 = 2x+(3y+8)$

3. What is the value of the digit 5 in the base-eight numeral 6543_{eight}?

   (a) 5
   (b) $5\cdot 64$
   (c) 500
   (d) $5\cdot 512$

4. Suppose $A = \{x, y, z\}$, $B \subseteq A$, and $n(B) = 3$. Which one of the following must be true?

   (a) $(x, x) \in A \times B$
   (b) $A \cap B = \emptyset$
   (c) $n(A \cup B) = 6$
   (d) $B - A = 3 - x - y - z$

   $B = \{x, y, z\}$

5. If m and n are both negative integers, then what can be said about $-m-n$?

   (a) $-m-n$ is a positive integer.
   (b) $-m-n$ is a negative integer.
   (c) $-m-n$ is not an integer.
   (d) There is not enough information to say anything about $-m-n$.

   $-(m+n)$ = opposite of a negative

6. Choose the multiplication model that best fits the following problem situation: *The graduates walked into the auditorium as a group in ten rows of four. How many graduates were there?*

   (a) Cartesian product
   (b) repeated addition
   (c) area/array
   (d) set partition

7. Convert the base-ten numeral 372 to base-four.

   (a) 11310_{four}
   (b) 78_{four}
   (c) 1131_{four}
   (d) 1032_{four}

$4^0 = 1,\ 4^1 = 4,\ 4^2 = 16,\ 4^3 = 64$

$4^4 = 256$

$4^4 = 256 \overline{)372}\ (1$ -256

$4^3 = 64 \overline{)116}\ (1$ -64

$4^2 = 16 \overline{)52}\ (3$ -48

$4^1 = 4 \overline{)4}\ (1$ -4

$0\ (0$

8. When using the 4-step, problem-solving process which one of these strategies would NOT be considered part of understanding the problem?

   (a) Reread the problem.
   (b) State the problem in your own words.
   (c) Determine what information is not needed.
   (d) Write an equation.

9. Suppose $A = 2^3 \cdot 5^2 \cdot 7 \cdot 13^3$ and $B = 2 \cdot 3 \cdot 5^3 \cdot 13^2$. Find the LCM of A and B.

   (a) $2 \cdot 3 \cdot 5^2 \cdot 7 \cdot 13^2$
   (b) $2^3 \cdot 3 \cdot 5^3 \cdot 7 \cdot 13^3$
   (c) $2 \cdot 5^2 \cdot 13^2$
   (d) $2^4 \cdot 3 \cdot 5^5 \cdot 7 \cdot 13^5$

10. Which one of these numbers is the 1371st term of the following arithmetic sequence?

    $18, 25, 32, 39, 46, 53, 60, 67, \ldots$

    $\vee\ \vee\ \vee\ \vee$

    $7\ \ 7\ \ 7\ \ 7\ \ \ldots$

    (a) 9608
    (b) 9615
    (c) 9601
    (d) 9597

$N^{TH}\ \text{TERM} = 7N + 11$

$7(1371) + 11 = 9608$

CONCLUSION BASED ON OBSERVATION

11. Which one of the following is an example of inductive reasoning?

(a) $2(3+5) = 2(5+3)$

(b) A sequence begins with 2,4,6,8. The next term must be 10.

(c) If $x = 10$, then $2x + 3 = 23$.

(d) Wednesdays are pizza days, so today is a pizza day.

12. Let W be the set of all whole numbers. The set A is defined below using set-builder notation. Which one of the given sets is equal to A?

$$A = \{x \mid x = 2n \text{ where } n \in W \text{ and } n < 2\}$$

N = 0 OR N = 1

(a) $\{0, 2\}$

(b) $\{0, 1\}$

(c) $\{0, 2, 4, 6, \ldots\}$

(d) $\{\ldots, -6, -4, -2, 0, 2\}$

13. Which one of the following is a valid test for divisibility by 20?

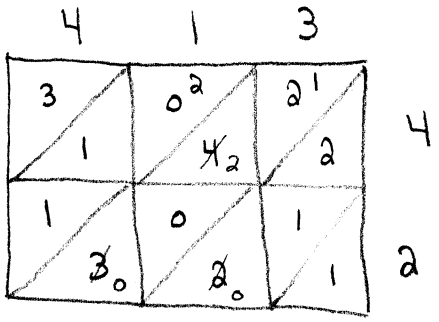
(a) An integer is divisible by 20 if and only if its last digit is 0.

(b) An integer is divisible by 20 if and only if the sum of its digits is divisible by 20.

(c) An integer is divisible by 20 if and only if it is divisible by both 2 and 10.

(d) An integer is divisible by 20 if and only if it is divisible by both 4 and 5.

14. (5 points) Use any multiplication algorithm to compute $413_{\text{five}} \times 42_{\text{five}}$.



3 4 0 0 1

34001 FIVE

Sum of digits is 54

15. (5 points) Test the number 749968830 for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, and 20. Show work and/or explain your reasoning.

2: Yes, ends in 0 & 2|0

3: Yes, sum of digits is 54 & 3|54

4: No, 4 ✗ 30

5: Yes, ends in 0

6: Yes, divisible by 2 & 3.

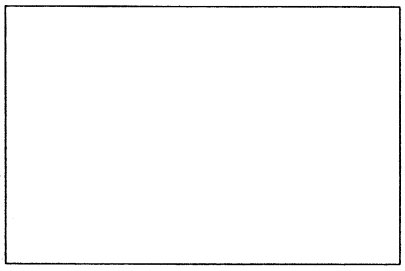
8: No, not divisible by 4

9: No, sum of digits is 54 & 9|54

10: Yes, ends in 0

20: No, not divisible by 4

16. (5 points) The area of a rectangle is 24 square inches. Its length and width are natural numbers. Use this information to find the rectangle with the least possible perimeter.



Area $= \ell \cdot w = 24$

Perimeter $= 2\ell + 2w$

Natural # factorizations of 24 are

1, 24 ; 2, 12; 3, 8; 4, 6 ← These pairs are possible dimensions.

Dimensions	Perimeter
1, 24 or 24, 1	2(1) + 2(24) = 50
2, 12 or 12, 2	2(2) + 2(12) = 28
3, 8 or 8, 3	2(3) + 2(8) = 22
4, 6 or 6, 4	2(4) + 2(6) = 20 ← Smallest perimeter

The 4 in by 6 in rectangle has least perimeter

17. (5 points)

(a) Find the fourth term of the geometric sequence whose first term is 3 and whose ratio is 5.

$3,\ 3\cdot 5,\ 3\cdot 5^2,\ 3\cdot 5^3$

$3,\ 15,\ 75,\ \boxed{375}$

(b) A recursive sequence is defined as follows:

$$B_1 = -2, \quad B_n = -3 \cdot B_{n-1} + 5, \text{ for } n = 2, 3, 4, \ldots$$

Find the third term of the sequence.

$B_2 = -3\cdot(-2) + 5 = 6 + 5 = 11$

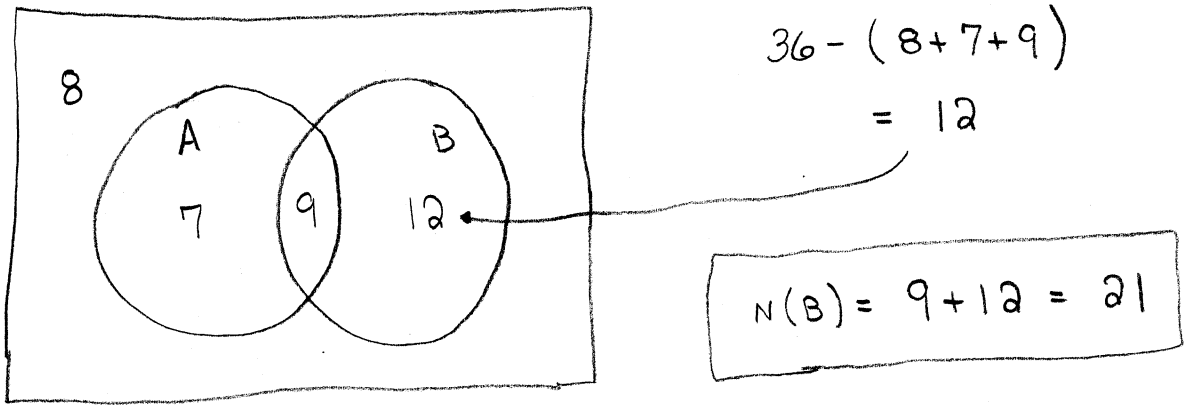
$B_3 = -3\cdot(11) + 5 = -33 + 5 = \boxed{-28}$

(c) Find the next term: $2, 4, 9, 17, 28, 42, \ldots$

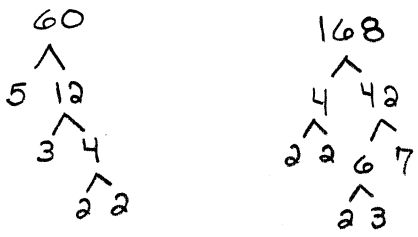
2 5 8 11 14 17

$42 + 17 = \boxed{59}$

18. (5 points) Suppose A and B are subsets of U, and U has 36 elements. Use a two-set Venn diagram to help you determine $n(B)$ if $n(A) = 16$, $n(A \cap B) = 9$, and $n(\overline{A \cup B}) = 8$.



19. (2 points) Find the greatest common divisor of 60 and 168.



$60 = 2^2 \cdot 3 \cdot 5$

$168 = 2^3 \cdot 3 \cdot 7$

$GCD = 2^2 \cdot 3 = 12$

20. (3 points) Use a model to illustrate and compute each product. Use a different model for each part. (Model what is given, not a related problem.)

(a) -3×4

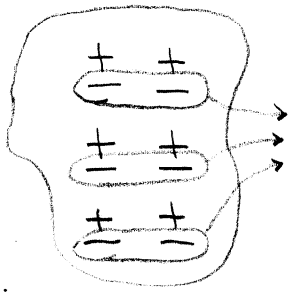
PATTERN

$3 \times 4 = 12$
$2 \times 4 = 8$
$1 \times 4 = 4$
$0 \times 4 = 0$

$-1 \times 4 = -4$
$-2 \times 4 = -8$
$-3 \times 4 = -12$

(b) $-3 \times (-2)$

CHARGES
① START WITH ZERO
② TAKE OUT 3 GROUPS OF -2



TAKE OUT

$-3 \times (-2) = +6$

21. (2 points) After looking at these examples:

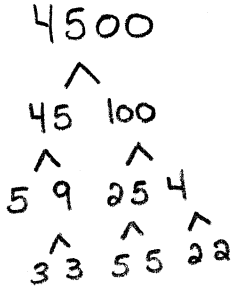
$$5 \cdot 3 + 5 \cdot 5 = 40, \quad 5 \cdot 2 + 5 \cdot 4 = 30, \quad 5 \cdot 8 + 5 \cdot 10 = 90,$$

Marcus conjectured that the sum of two multiples of 5 is a multiple of 10. Is he correct? If not, give a counterexample.

NO, HE IS WRONG.

$3 \cdot 5 + 2 \cdot 5 = 15 + 10 = 25$ ← NOT A MULTIPLE OF 10.

22. (5 points) Find the prime factorization of 4500. Then use your factorization to determine the number of positive integer divisors of 4500.



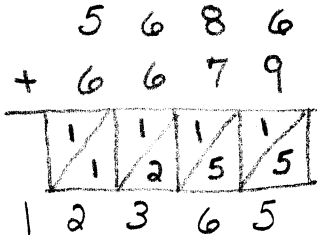
$4500 = 2^2 \cdot 3^2 \cdot 5^3$

NUMBER OF DIVISORS IS

$3 \cdot 3 \cdot 4 = 36$

23. (5 points) Use a NONSTANDARD algorithm to compute each of the following.

(a) $5686 + 6679$ LATTICE ADDITION



12,365

(b) $345 - 269$

EQUAL ADDITIONS

$345 + 1 \rightarrow 346 + 30 \rightarrow 376$

$-269 + 1 \rightarrow -270 + 30 \rightarrow -300$

76

76

(c) $6745 \div 5$

SHORT DIVISION

$5\overline{)6^{1}7^{2}4^{4}5}$ $\quad 1349$

1349

24. (2 points) State a basic property of the Hindu-Arabic numeration system.

ALL NUMERALS ARE CONSTRUCTED FROM 10 BASIC DIGITS: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

25. (1.5 points) Write an application problem (i.e. a word problem) involving subtraction in which the subtraction is best modeled using comparison.

THERE ARE FIVE DESKS IN THE 1ST ROW AND THREE DESKS IN THE 2ND ROW. HOW MANY MORE DESKS ARE IN THE 1ST ROW? $5 - 3 = 2$

26. (1.5 points) Write the expanded form of the number 54967.

$5 \times 10^4 + 4 \times 10^3 + 9 \times 10^2 + 6 \times 10^1 + 7 \times 10^0$

27. (2 points) Use an integer subtraction model to illustrate and compute $-2 - (-5)$.

NUMBER LINE...

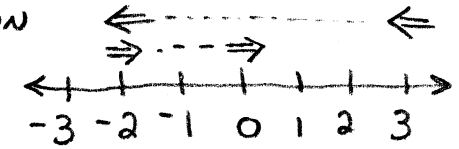
① START AT ZERO FACING RIGHT.

② GO BACKWARDS 2 UNITS TO MODEL -2

③ TURN AROUND TO MODEL SUBTRACTION

④ GO BACKWARDS 5 UNITS TO MODEL -5

⑤ END AT $+3$



$-2 - (-5) = 3$