

Math 200 - 2nd Final Exam

May 16, 2011

Name key
Score _____

Show all work to receive full credit. Supply explanations where necessary. Multiple choice problems are worth 0, 1, or 2 points depending on your answer and the work shown.

1. A student said that all odd numbers are prime. Which one of the following is a counterexample?

(a) 2

(b) 7

(c) 9 ← ODD THAT'S NOT PRIME.

(d) 12

2. Compute the sum: $1 + 2 + 3 + 4 + \dots + 1723 + 1724 + 1725$

(a) 1,488,675

(b) 2,977,350

(c) 5,182

(d) 1,487,812.5

$$\frac{1726 \times 1725}{2} = 1488675$$

3. Which one of the following sets is NOT well defined?

(a) The set of all natural numbers

(b) The set of all integers less than -12

(c) The set of all whole numbers less than 0

(d) The set of all big integers WHAT IS BIG?

4. The first term of a **geometric series** is 4 and its ratio is 7. Which one of the following is its third term?

(a) 1372

(b) 196

(c) 18

(d) 25

$$4, 4 \times 7, 4 \times 7 \times 7$$

↘ 196

5. If $A = \{2, 4, 6\}$ and $B = \{1, 3\}$, then which one of the following is $n(A \times B)$?

(a) 6

(b) (1, 6)

(c) $\{1, 2, 3, 4, 6\}$

(d) 5

↑
NUMBER OF
ELEMENTS IS
 $3 \times 2 = 6$

6. Choose the multiplication model that best fits the following problem situation: A restaurant offers combo meals with choices of 10 different entrees and 6 different appetizers. How many different combo meals are offered?

- (a) array
- (b) missing factor
- (c) repeated addition
- (d) Cartesian product

NUMBER OF COMBINATIONS ?

7. Which one of the following is a fraction that is equivalent to $36/6$ but has a numerator of 18?

- (a) 6
- (b) $108/18$
- (c) $18/3$
- (d) $18/12$

8. Which one of these is NOT a correctly written base-seven numeral?

- (a) 66006_{seven}
- (b) 1110011_{seven}
- (c) 5_{seven}
- (d) 17_{seven}

9. Let Z be the set of integers. Which one of the following is NOT an element of the set $A = \{x \mid x = 3n + 1 \text{ where } n \in Z\}$.

- (a) 10
- (b) -6
- (c) -2
- (d) 7

↑ ELEMENTS ARE MULTIPLES OF 3 PLUS 1

10. Compute the sum: $321_{\text{five}} + 113_{\text{five}} + 24_{\text{five}}$

- (a) 1013_{five}
- (b) 458_{five}
- (c) 513_{five}
- (d) 423_{five}

$$\begin{array}{r}
 1 1 \\
 3 2 1 \\
 \cancel{X_0} 1 3 \\
 \cancel{X_1} \cancel{X_3} \\
 \hline
 2 1 0 1 3
 \end{array}$$

BASE FIVE

11. Choose the division model that best fits the following problem situation: *Ms. Smith has 24 markers that she must divide into 6 groups of equal size. How many markers will there be in each group?*

- (a) missing factor
- (b) repeated subtraction
- (c) Cartesian product
- (d) set partition

How many in each group?



12. If x is a negative number, then what can be said about $-2x$?

- (a) $-2x$ is negative
- (b) $-2x$ is positive
- (c) $-2x$ could be zero
- (d) More information is needed

↑ OPPOSITE OF A NEGATIVE

13. Suppose $A = \{x, y, z\}$ and $A \sim B$. Which one of the following must be true?

- (a) $x \in B$
- (b) $n(B) = 3$
- (c) $B = \bar{A}$
- (d) $A \cup B = A$

↑ EQUIVALENT. B MUST HAVE 3 ELEMENTS.

14. Which one of the following facts demonstrates the associative property of multiplication?

- (a) $2(x + 1) + 4 \cdot 5 = 2(x + 1) + 5 \cdot 4$ Comm.
- (b) $(6x + 10) = 2(3x + 5)$ Dis.
- (c) $2 \cdot (6 \cdot 7) + 1 = (2 \cdot 6) \cdot 7 + 1$ Assoc
- (d) $(4 + 1) + 2 = 4 + (1 + 2)$ Assoc prop of addition

15. Which one of the following divisibility tests is incorrect?

- (a) A whole number is divisible by 3 if the sum of its digits is divisible by 3.
- (b) A whole number is divisible by 6 if is divisible by both 2 and 3.
- (c) A whole number is divisible by 8 if the number formed by its last three digits is divisible by 8.
- (d) A whole number is divisible by 9 if the number formed by its last 2 digits is divisible by 9.

16. (5 points) Clearly state the steps of the problem-solving process (in order). Then choose any one step and state two different strategies associated with that step.

① UNDERSTAND THE PROBLEM

② DEVISE A PLAN

③ CARRY OUT THE PLAN

④ LOOK BACK

i) CHECK

ii) GENERALIZE

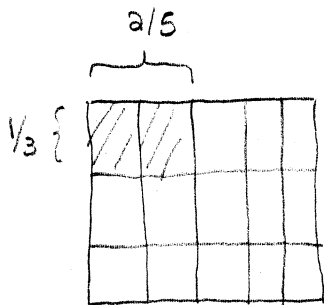
17. (5 points) Use a model to illustrate and compute each product. (Model what is given, not a related problem.)

(a) $2 \times \frac{3}{7}$ REPEATED ADDITION

$$2 \times \frac{3}{7} = \frac{3}{7} + \frac{3}{7} = \frac{6}{7}$$

(b) $\frac{1}{3} \times \frac{2}{5}$

AREA/ARRAY



$\frac{2}{15}$ IS SHADED

$$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$$

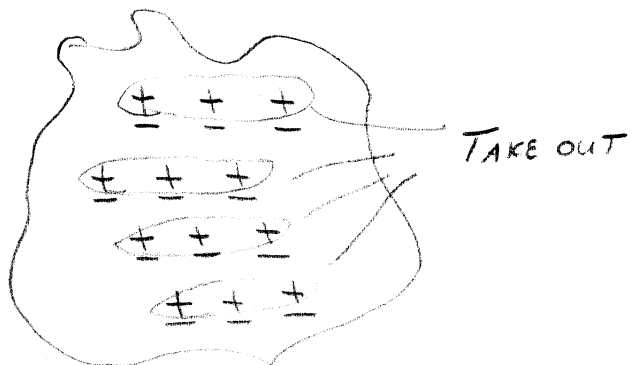
(c) -4×3 CHARGES

① START WITH ZERO

② TAKE OUT 4 groups OF 3+'s

③ LEAVES 12-'s

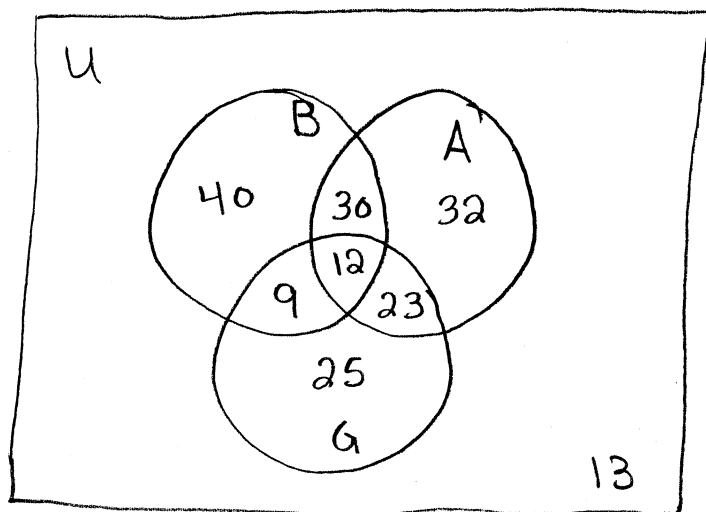
$$-4 \times 3 = -12$$



18. (5 points) 184 children were asked to name the fruits they often eat. The following results were obtained:

- 91 said bananas
- 97 said apples
- 69 said grapes
- 42 said bananas and apples
- 21 said bananas and grapes
- 35 said apples and grapes
- 12 said bananas, apples, and grapes

Organize this data in a three-set Venn diagram. How many children surveyed named none of these three fruits?



$$40 + 30 + 32 + 9 + 12 + 23 + 25 = 171$$

$$184 - 171 = \underline{\underline{13}}$$

13 NAMED NONE

19. (5 points) Use any method to find both the GCD and LCM of 630 and 6615.

$$6615 \div 630 = 10 \text{ r } 315$$

$$630 \div 315 = 2 \text{ r } 0$$

$$\left. \begin{array}{l} 6615 \div 630 = 10 \text{ r } 315 \\ 630 \div 315 = 2 \text{ r } 0 \end{array} \right\} \rightarrow \text{GCD}(6615, 630) = \text{GCD}(630, 315) = 315$$

$$\text{LCM} = \frac{630 \times 6615}{315} = 13,230$$

GCD is 315, LCM is 13230

20. (5 points) Carefully explain how you would efficiently determine whether 839 is prime? Is it?

$$\sqrt{839} \approx 28.97$$

To DETERMINE IF 839 IS PRIME, YOU MUST CHECK IT FOR DIVISIBILITY BY ALL PRIMES LESS THAN 28:

2, 3, 5, 7, 11, 13, 17, 19, 23.

839 IS NOT DIVISIBLE BY ANY OF THESE. IT IS PRIME.

21. (5 points) Write the numeral 353_{six} in expanded form. Then list the next five base-six numerals.

$$3 \times 6^2 + 5 \times 6^1 + 3 \times 6^0$$

① 353_{six} , ② 354_{six} , ③ 355_{six} , ④ 400_{six} , 401_{six} ,

402_{six}

⑤

22. (5 points) Clearly state the rule for adding two integers with opposite signs. Give an example that illustrates your rule.

SUBTRACT THE ABSOLUTE VALUES, LEAST FROM GREATEST.

THEN GIVE THE RESULT THE SIGN OF THE ORIGINAL

ADDED WITH THE GREATEST ABSOLUTE VALUE.

Ex

$$-5 + 3 = -(5 - 3) = -2$$

23. (5 points) Suppose U is the set of all Americans, A is the set of all American smokers, and B is the set of all Americans with health problems. Describe a person who is an element of each of the following sets.

(a) $A \cap B$

AN ELEMENT IS AN AMERICAN SMOKER
WITH HEALTH PROBLEMS.

(b) $A \cap \bar{B}$

AN ELEMENT IS AN AMERICAN SMOKER
WITH NO HEALTH PROBLEMS.

(c) $B - A$

AN ELEMENT IS AN AMERICAN WITH HEALTH
PROBLEMS WHO IS NOT A SMOKER.

24. (5 points) Write a whole number with five different digits. Then use divisibility tests to test for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, and 11.

My number is:

43782

Sum of digits is $4+3+7+8+2 = 14$

By 2: Yes, BECAUSE $2 \mid 2$

By 3: No, BECAUSE $3 \nmid 14$

By 4: No, BECAUSE $4 \nmid 82$

By 5: No, BECAUSE $2 \nmid 5$

By 6: No, BECAUSE NOT DIVISIBLE BY 3.

By 8: No, BECAUSE $8 \nmid 782$ ($782 \div 8 = 97.75$)

By 9: No, BECAUSE $9 \nmid 14$

By 10: No, BECAUSE $10 \nmid 2$

By 11: No, BECAUSE $11 \nmid 2$

$$(4+7+2) - (3+8) = 2$$