Name __key__

February 25, 2015   Score _____

Show all work. Supply explanations when necessary.

1. (10 points) Let $f(x) = x\left[\sqrt{x+1} - \sqrt{x}\right]$.

  (a) For which x-values would you expect to encounter a loss of significance when computing $f(x)$?

FOR LARGE VALUES OF X, $\sqrt{x+1} \approx \sqrt{x}$,

AND THE COMPUTATION OF $f(x)$ INVOLVES

SUBTRACTION OF NEARLY EQUAL QUANTITIES.

  (b) Use six-digit arithmetic with rounding to compute $f(100)$. By comparing your result with your calculator's (full precision) result, determine how many digits in your computation are correct.

SIX-DIGIT...   CALCULATOR'S VALUE...

$f(100) = 100(10.0499 - 10)$ 4.98756

$ = \boxed{4.99000}$

ONLY TWO DIGITS ARE CORRECT IN THE SIX-DIGIT COMPUTATION.

  (c) Rewrite the expression for $f(x)$ in such a way that loss of significance is avoided.

$$f(x) = \frac{x\left[\sqrt{x+1} - \sqrt{x}\right]\left[\sqrt{x+1} + \sqrt{x}\right]}{1 \qquad \sqrt{x+1} + \sqrt{x}} = \frac{x(x+1-x)}{\sqrt{x+1} + \sqrt{x}} = \boxed{\frac{x}{\sqrt{x+1} + \sqrt{x}}}$$

  (d) Using six-digit arithmetic with rounding, recompute $f(100)$ with your new expression. Determine how many digits are correct.

SIX-DIGIT...

$$\frac{100}{10.0499 + 10} = \frac{100}{20.0499}$$

$$= \boxed{4.98756}$$

ALL SIX DIGITS ARE CORRECT (WHEN COMPARED TO CALCULATOR).

1

2. (3 points) The value of π is given by the series

$$4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

Use the first seven terms of this series to approximate π and find the relative error in your approximation.

$$4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13}\right) = 3.283738\ldots$$

$$\text{REL ERROR} = \left|\frac{\text{COMPUTED VALUE} - \pi}{\pi}\right| = 0.045246\ldots$$
$$\approx 4.5\%$$

3. (2 points) Notice that $\cos(1000000000001\pi) = -1$. However, if you attempt to evaluate $\cos(1000000000001\pi)$ on your TI-83/84 calculator, you will get a domain error. Explain why your calculator cannot compute this value.

BECAUSE THE TI-83/84 USE 13-DIGIT ARITHMETIC, THE CALCULATOR CAN'T TELL THE DIFFERENCE BETWEEN BIG NUMBERS WHOSE FIRST 13 DIGITS MATCH. YOUR CALCULATOR "BELIEVES" 1000000000001π IS THE SAME AS 1000000000000π, BUT THESE NUMBERS HAVE VERY DIFF. COSINES.

4. (2 points) Give an example of an expression that makes use of the operators <=, ||, and !=, and has a value of one (true).

$$\left(5 <= 3\right) \;||\; \left(1 \;!= 2\right)$$

FALSE   OR   TRUE $\longrightarrow$ TRUE

5. (2 points) Give an example of an expression that makes use of the operators ==, &&, and !, and has a value of zero (false).

$$\left(7 == 8\right) \;\&\&\; !\left(7 > 4\right)$$

FALSE   AND   NOT TRUE $\longrightarrow$ FALSE

6. (1 point) By default, how many significant digits are displayed in a C++ program?

6 DIGITS

7. (2 points) A function has the following header line:

```
int func( float x )
```

Could such a function return the value 13.5? Explain.

No, THIS FUNCTION RETURNS AN INT TYPE

8. (6 points) Suppose a, b, and c have all been declared integers and assigned the common value of 3. What will the values of all three variables be after each statement is executed.

(a) a += (b + c)

$a = a + (b + c)$

$a = 9$
$b = 3$
$c = 3$

(b) a = (++b) + (c++)

$a = 4 + 3$

$a = 7$
$b = 4$
$c = 4$

(c) a = (b != ++c)

$a = (3 \ne 4)$

TRUE

$a = 1$
$b = 3$
$c = 4$

9. (2 points) Determine the output of the following program. Explain.

```cpp
#include <iostream>
using namespace std;

int main()
{
   float x = 2e+20, y = 3e+30;
   cout << x * y + 1.0f;

   return( 0 );
}
```

THE OUTPUT WILL BE

inf BECAUSE X*y WILL

RESULT IN OVERFLOW.

X*y IS TOO BIG

FOR FLOATS.

10. (2 points) Give an example of a header line for a function that returns a Boolean type and accepts two arguments: an integer type and a double type.

$$bool\ func(int\ x,\ double\ y)$$

11. (3 points) Determine the exact output of the following program.

```cpp
#include <iostream>
using namespace std;

int main()
{
    int i;
    i = 12.7;

    if ( i > 12 ) {
       cout << 5 + (i++) << endl;
    }
    else {
       cout << "Have a great\n day!" << endl;
    }

    return( 0 );
}
```

Have a great
day!
(LINE BREAK)
———————————
(RETURN TO OS)

12. (3 points) Explain how and why one would use the keyword const.

USE const AS PART OF A DECLARATION AND DEFINITION

e.g. const float x = 1.59375;

THIS WILL NOT ALLOWED X TO BE CHANGED IN THE PROGRAM,

13. (3 points) Write a for statement for the following case: AND you'll GET A COMPILE-

Use a counter named icnt that has an initial value of 10, a final value of 45,   TIME ERROR

and an increment of 5.   IF you TRY.

int icnt;

for (icnt = 10; icnt <= 45; icnt += 5)

4

14. (4 points) Find four compile-time errors in the following source code.

```cpp
#include <iostream>
using namespace std;

int main( )
{
    const float _E_ = 2.71828;
    cout << sin( _E_ ) << endl;

    double num;
    3.1415926 = num;

    _E_ += 10;

    return( 0 );
}
```

[Handwritten annotations:]
← SIN IS NOT DEFINED
SINCE <CMATH> IS NOT
INCLUDED
A CONSTANT CANNOT BE ASSIGNED A VALUE.
A CONST TYPE CANNOT BE CHANGED
MISSING TERMINATOR

15. (5 points) Complete the following C++ program, which uses a `for` loop to display the values of $\sin(0.1), \sin(0.2), \ldots, \sin(1.0)$. (Each value is to be displayed on its own line.)

```cpp
#include <iostream>
#include <cmath>
using namespace std;

int main( )
{
    for( int i = 1; i <= 10; i++ )
        cout << sin( 0.1 * i ) << endl;

    return( 0 );
}
```

16. (Extra Credit 2 pts) Suppose you want each value in your program above to be displayed with at most 4 significant digits. How can this be accomplished?

[Handwritten answer:] cout.precision(4);