

Math 206 - 1st Final Exam

December 1, 2010

Name key Score _____

Show all work. Supply explanations where necessary. Use only a compass and a straightedge for constructions. For each construction, the steps you follow must be apparent.

1. (3 points) An experiment consists of simultaneously rolling a 4-sided die and flipping a quarter.

- (a) List the elements of an equally-likely sample space.

$$S = \{ 1H, 2H, 3H, 4H, 1T, 2T, 3T, 4T \}$$

EACH OUTCOME HAS PROB $\frac{1}{8}$.

- (b) Referring to your sample space, what is the event of rolling a 2?

$$E = \{ 2H, 2T \}$$

- (c) What is the probability of rolling a 1 or flipping a head?

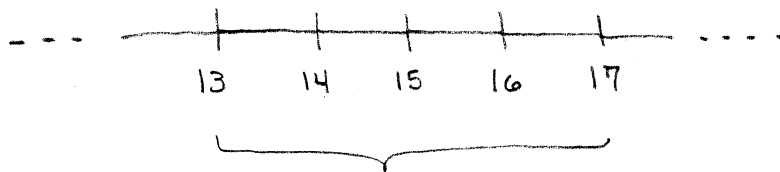
$$E = \{ 1H, 1T, 2H, 3H, 4H \}$$

$$P(E) = \frac{5}{8}$$

- (d) Is your probability in part (c) an experimental probability or a theoretical probability?

THEORETICAL, IT WAS ASSIGNED
ASSUMING EACH OUTCOME IS
EQUALLY LIKELY.

2. (2 points) A ladybug lands in a random location on a meter stick. What is the probability that the bug lands between the 13 cm mark and the 17 cm mark?

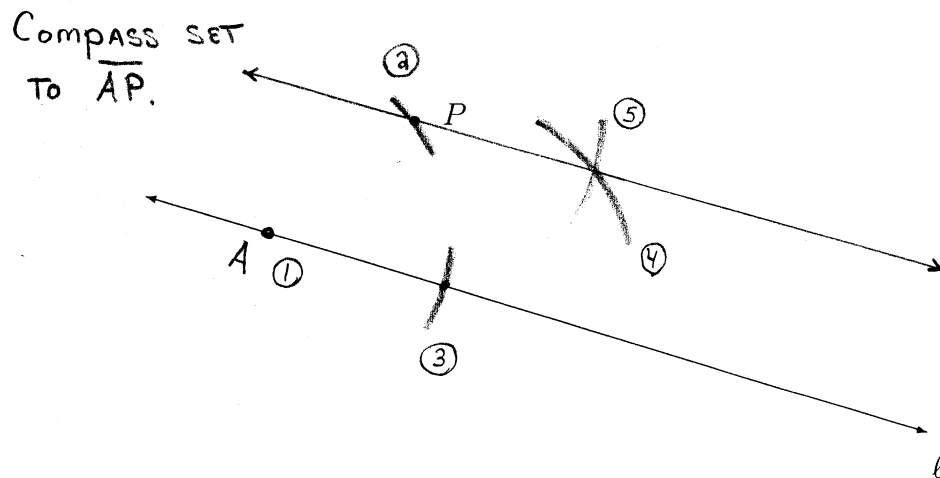


4 cm ON METER STICK

⇒

$$\text{Prob is } \frac{4}{100} = \frac{1}{25}$$

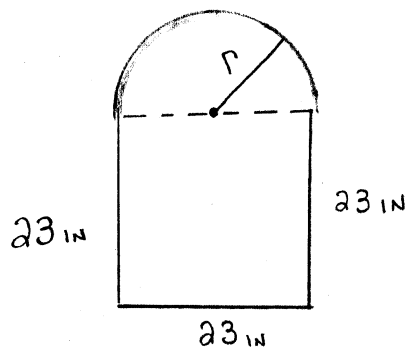
3. (5 points) Use only a compass and straightedge to construct a line through P parallel to l .



4. (5 points) A semicircle fits perfectly on top of a square. If the square has sides of length 23 in, find the distance around the outside of the figure. Write your final answer in meters, rounded to the nearest hundredth.

$$r = \frac{23}{2} = 11.5 \text{ in}$$

$$\begin{aligned} \text{Perimeter} &= 23 + 23 + 23 \\ &\quad + \frac{1}{2} (2\pi 11.5) \\ &= 105.1283155 \text{ in} \end{aligned}$$



$$\frac{105.1283155 \text{ in}}{1} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 267.026 \text{ cm}$$

2.67 m

5. (6 points) The organizers of a charity event have planned the following game:

Contestants will pay \$5 to randomly select a bill from a box containing one \$100 bill, two \$20 bills, two \$10 bills, five \$5 bills, and ninety \$1 bills. \Rightarrow 100 BILLS TOTAL.

(a) On average, how much should the event organizers expect to make for each person who plays the game?

$$P(\$100) = \frac{1}{100}, \quad P(\$20) = \frac{2}{100}, \quad P(\$10) = \frac{2}{100}$$

$$P(\$5) = \frac{5}{100}, \quad P(\$1) = \frac{90}{100}$$

$$\begin{aligned} \text{EXPECTED VALUE} &= (\$100) \left(\frac{1}{100}\right) + (\$20) \frac{2}{100} + (\$10) \left(\frac{2}{100}\right) \\ &+ (\$5) \left(\frac{5}{100}\right) + (\$1) \left(\frac{90}{100}\right) = \$2.75 \end{aligned}$$

(b) Is the game fair? Explain.

No,

MAKE \$2.25 PER PLAYER

$$\text{COST} = \$5 \neq \text{EXPECTED VALUE} = \$2.75$$

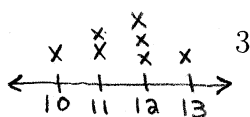
(c) What are the odds against selecting a \$5 bill?

$$\text{PROB IS } \frac{5}{100} \Rightarrow \text{ODDS AGAINST} = \frac{95}{5} = \frac{19}{1}$$

6. (3 points) What is the difference between a line plot and a line graph? Give an example of a situation in which you would use each type of graph.

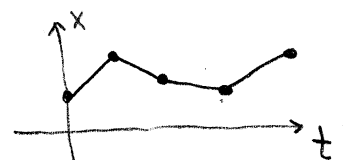
A LINE PLOT IS ANALOGOUS TO A FREQUENCY TABLE.

FREQUENCY OF DATA VALUES ARE SHOWN BY PLACING DOTS ABOVE THE VALUE ON A NUMBER LINE. COULD BE USED FOR TEST SCORES TO SCORES & FREQS.

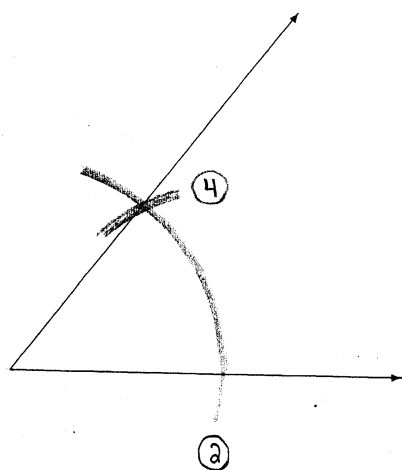


A LINE GRAPH IS USUALLY USED TO SHOW TRENDS OVER TIME, SUCH AS THE PRICE OF A STOCK OVER TIME.

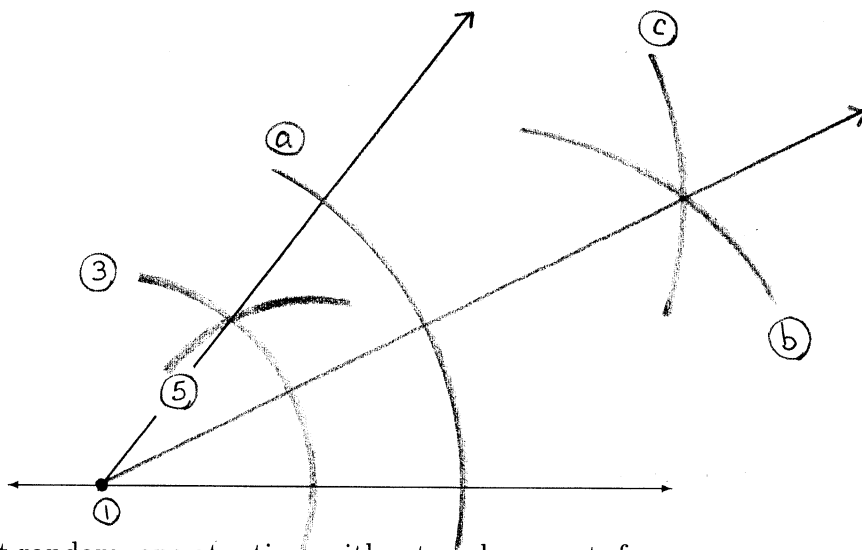
ORDERED PAIRS ARE PLOTTED AND CONNECTED WITH LINE SEGMENTS.



7. (6 points) Copy the given angle so that its initial side lies on the given line. Then bisect the copied angle.



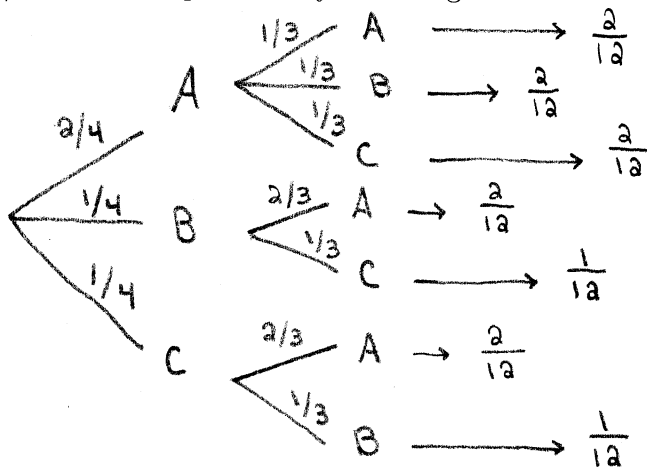
1-5 TO COPY \angle .
A-C TO BISECT



8. (6 points) Two letters are selected at random, one at a time without replacement, from the following box.

A A B C

- (a) Sketch the probability tree diagram associated with this two-stage experiment.



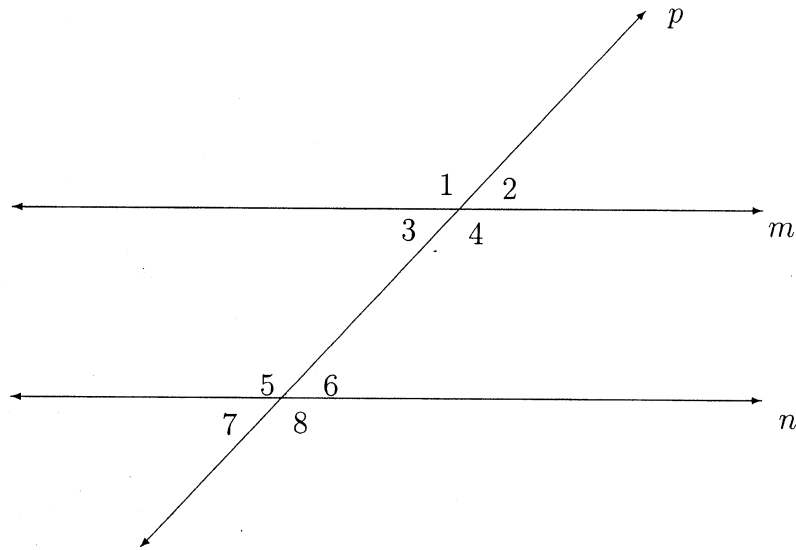
- (b) What is the probability that one of the selected letters is a C?

$$AC, BC, CA, CB \rightarrow \frac{2}{12} + \frac{1}{12} + \frac{2}{12} + \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$$

- (c) What are the odds in favor of drawing a C?

$$\text{Prob} = \frac{1}{2} \Rightarrow \text{ODDS ARE } \frac{1}{1}$$

9. (6 points) In the following figure, the parallel lines m and n are being cut by transversal p .



- (a) Name a pair of alternating exterior angles.

$$\angle 7 \text{ \& } \angle 2 \quad \text{or} \quad \angle 8 \text{ \& } \angle 1$$

- (b) Name a pair of corresponding angles.

$$\angle 1 \text{ \& } \angle 5, \quad \angle 3 \text{ \& } \angle 7, \quad \angle 2 \text{ \& } \angle 6, \quad \text{or} \quad \angle 4 \text{ \& } \angle 8$$

- (c) Name a pair of alternating interior angles.

$$\angle 3 \text{ \& } \angle 6 \quad \text{or} \quad \angle 5 \text{ \& } \angle 4$$

- (d) Name a pair of vertical angles.

$$\angle 1 \text{ \& } \angle 4, \quad \angle 2 \text{ \& } \angle 3, \quad \angle 6 \text{ \& } \angle 7, \quad \angle 5 \text{ \& } \angle 8$$

- (e) Name a pair of adjacent angles.

$$\angle 1 \text{ \& } \angle 2, \quad \angle 2 \text{ \& } \angle 4, \quad \angle 4 \text{ \& } \angle 3, \quad \dots$$

- (f) Name a pair of supplementary angles that are not adjacent.

$$\angle 1 \text{ \& } \angle 7, \quad \angle 1 \text{ \& } \angle 6, \quad \angle 2 \text{ \& } \angle 8, \quad \angle 2 \text{ \& } \angle 5, \quad \dots$$

10. (5 points) Last year the junior high spirit squad sold items as homecoming souvenirs. In all, they sold 61 pom-poms for \$1 each, 57 pins for \$1 each, 19 cups for \$2 each, 45 key rings for \$3 each, and 3 T-shirts for \$22 each.

(a) Find the mean, median, and mode(s) of the amounts collected for homecoming souvenirs.

61 - \$1, 57 - \$1, 19 - \$2, 45 - \$3, 3 - \$22

↑ MEDIAN & MODE ARE CLEARLY \$1 ⇒

MEDIAN = \$1
MODE = \$1

$$\text{MEAN} = \frac{61(1) + 57(1) + 19(2) + 45(3) + 3(22)}{61 + 57 + 19 + 45 + 3} = \frac{357}{185}$$

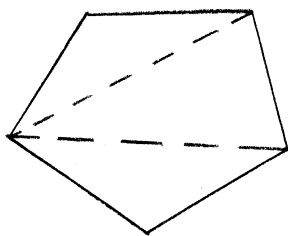
≈ \$1.93

(b) Which measure of central tendency is most appropriate when describing the average price paid for a souvenir? Why?

IN THIS CASE, THE \$1 PRICE IS THE MOST APPROPRIATE MEASURE OF CENTER.

THE MEAN (\$1.93) DOES NOT REPRESENT THE "AVERAGE" COST PER ITEM. NEARLY 64% OF ITEMS

11. (3 points) Sketch a convex pentagon. What is the sum of the interior angles? Explain how your result can be derived by using triangles. SOLD FOR EXACTLY \$1.



THE INTERIOR REGION CAN BE DECOMPOSED INTO 3 TRIANGLES, WHOSE INTERIOR ∠'S ADD UP TO 180° FOR EACH.

PENTAGON → 180° × 3 = 540°

12. (5 points) Which of the following could be used as a definition of a square? Circle all that apply. Explain your reasoning to get partial credit.

(a) A square is a rhombus with a right angle.

(b) A square is a quadrilateral with four congruent sides.

← RHOMBUS BUT NOT NECESSARILY A SQUARE.

(c) A square is a rectangle that is also a kite.

(d) A square is a kite with a right angle.



(e) A square is a rectangle that is also a rhombus.

(f) A square is a parallelogram that is also a kite.

← RHOMBUS BUT NOT NECESSARILY A SQUARE.

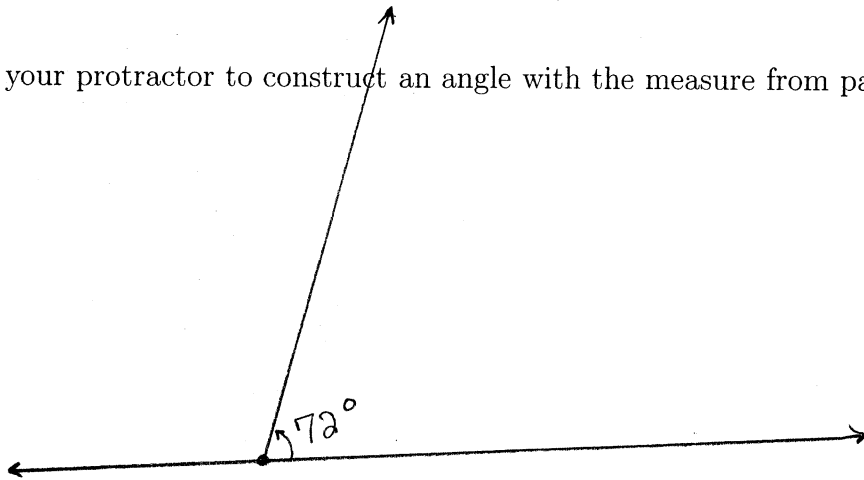
13. (4 points) Stacy makes \$4000 each month, and \$800 of that goes to paying her mortgage.

(a) If Stacy was to build a circle graph (pie chart) showing her monthly expenditures, what would be the measure of the angle of the portion of the circle graph corresponding to her mortgage?

$$\frac{800}{4000} = 0.2 = 20\%$$

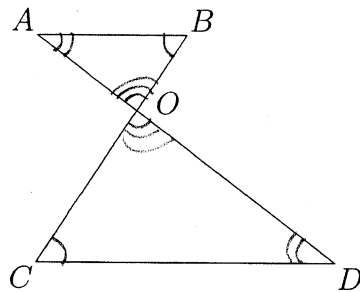
$$20\% \text{ of } 360^\circ = \boxed{72^\circ}$$

(b) Use your protractor to construct an angle with the measure from part (a).



14. (5 points) Given that $AB \parallel CD$, prove that $\triangle AOB$ is similar to $\triangle DOC$. Carefully explain your reasoning.

① \overline{AB} AND \overline{CD} ARE PARALLEL LINES CUT BY TRANSVERSAL \overline{BC} . ALT INTERIOR \angle 'S ARE CONGRUENT
 $\Rightarrow \angle ABC \cong \angle DCB$
 (AS MARKED).



③ VERTICAL \angle 'S ARE CONGRUENT
 $\Rightarrow \angle AOB \cong \angle DOC$
 (AS MARKED)

④ $\triangle AOB \sim \triangle DOC$
 By AAA

② SAME REASON WITH DIFFERENT TRANSVERSAL \overline{AD}
 $\Rightarrow \angle BAD \cong \angle ADC$
 (AS MARKED)

15. (5 points) Fill in the blank with the correct word or phrase.

(a) A simple, closed, polygonal curve is called a(n) POLYGON

(b) A(n) ACUTE angle is an angle whose measure lies between 0° and 90° .

(c) CPCTC stands for CORRESPONDING PARTS OF CONGRUENT TRIANGLES ARE CONGRUENT.

(d) A polygon for which all sides are congruent and all interior angles are congruent is called a(n) REGULAR polygon.

(e) If the sum of the measures of two angles is 90° , then the angles are said to be COMPLEMENTARY

16. (2 points) Given the figure below and no other information, which of the following statements is not necessarily true?

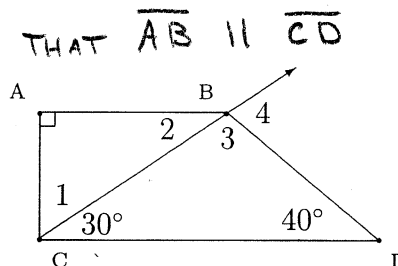
(a) $m(\angle 3) = 110^\circ$

(b) $m(\angle 1) = 60^\circ$ WE DON'T KNOW THAT $\overline{AB} \parallel \overline{CD}$

(c) $m(\angle 1) + m(\angle 2) = 90^\circ$

(d) $70^\circ + m(\angle 3) = 180^\circ$

(e) $\angle 1$ is an acute angle.



17. (4 points) Consider the following collection of test scores.

98 52 98 45
37 45 98 79

(a) Compute the mean and standard deviation. You may use your calculator to compute the standard deviation.

$$\text{MEAN} = \frac{98 + 52 + 98 + 45 + 37 + 45 + 98 + 79}{8} = \boxed{69}$$

From CALC, $\sigma = \boxed{25.22}$

(b) Compute the z-score corresponding to 88.

$$Z = \frac{88 - 69}{25.22} \approx 0.75$$