Math 206 - 2nd Final Exam

December 13, 2010

Name key

Score _____

Show all work. Supply explanations where necessary. Use only a compass and a straightedge for constructions. For each construction, the steps you follow must be apparent.

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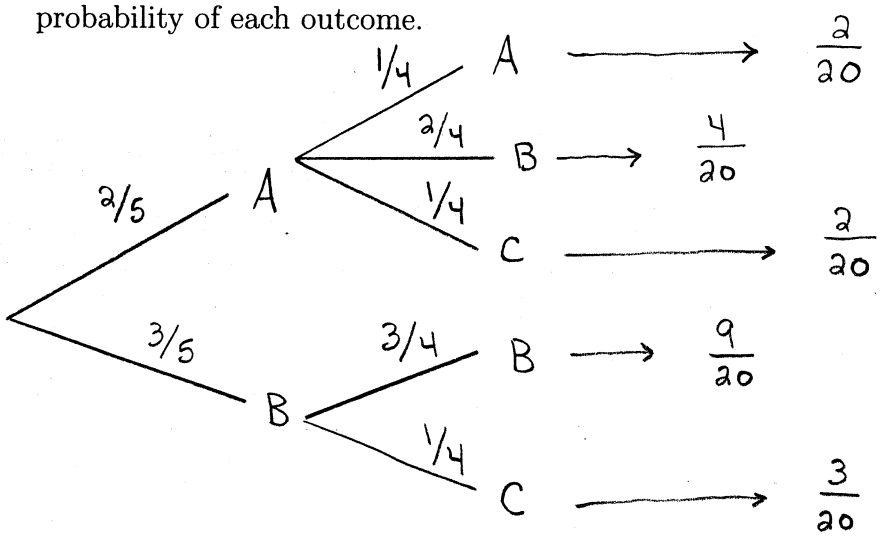
1. (5 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected from the second box.

A A B B B		B B C

   (a) Sketch the probability tree associated with this two-stage experiment and find the probability of each outcome.



   (b) Are the probabilities above theoretical or experimental? Explain your reasoning.

THEORETICAL. WE DID NOT ACTUALLY DO THE EXPERIMENT AND ASSIGN PROBABILITIES BASED ON OBSERVATION. WE ASSIGNED PROBABILITIES BY ASSUMING EACH SELECTION WAS EQUALLY LIKELY AT EACH STAGE.

2. (3 points) Ms. Ridder teaches two algebra classes. Section 1 has 17 students, and their mean score on the recent test was 72.5. Section 2 has 24 students, and their mean score was 68.25. Find the mean score of the combined classes.

$$\frac{17(72.5) + 24(68.25)}{17+24} \approx 70.012$$

3. (6 points) The numbers shown below are the heights (in centimeters) of the children in Mr. Strand's class. Compute the median, quartiles, IQR, and outlier cut-offs. Then construct the corresponding box plot. (Use the attached graph paper for the box plot.)

~~121~~, ~~120~~, ~~121~~, ~~128~~, ~~124~~, ~~126~~, ~~160~~,
~~132~~, ~~140~~, ~~140~~, ~~152~~, ~~150~~, ~~142~~, ~~132~~

120, 121, 121, 124, 126, 128, 132, 132, 140, 140, 142, 150, 152, 160

Q_1 $\quad$ Q_3

$\text{MED} = \frac{132+132}{2} = 132$

MED = 132

$Q_1 = 124$

$Q_3 = 142$

IQR = 142 − 124 = 18

$1.5 \times IQR = 1.5 \times 18 = 27$

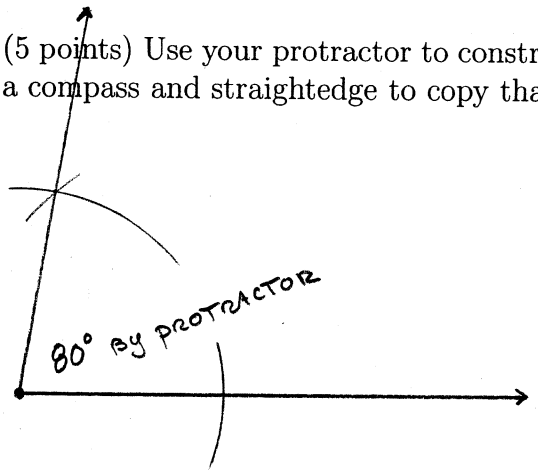
CUTOFFS: OUTLIERS ARE ...

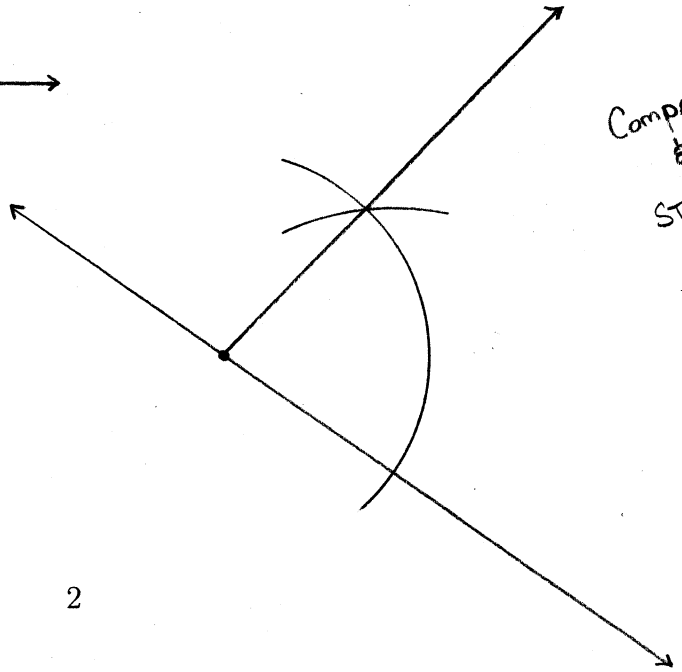
LESS THAN $124 - 27 = 97$

GREATER THAN $142 + 27 = 169$

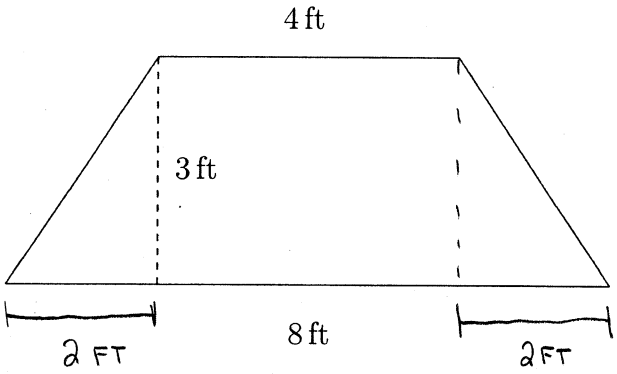
THERE ARE NO OUTLIERS.
SEE GRAPH PAPER.

4. (5 points) Use your protractor to construct an angle that measures 80°. Then use only a compass and straightedge to copy that angle to a new location.





5. The figure shown below is an isosceles trapezoid.



(a) (2 points) Find the area of the figure.

$$\text{Area} = \frac{1}{2}(4+8)(3) = \frac{1}{2}(12)(3) = 18$$

$$\boxed{18 \text{ FT}^2}$$

(b) (6 points) Find the perimeter of the figure. Write your final result in meters, rounded to two decimal places. (Hint: You will need the Pythagorean Theorem to find the slant length.)

$2^2 + 3^2 = 4 + 9 = 13$

Slant length $= \sqrt{13}$ FT

$$\text{Perimeter} = \sqrt{13} + 4 + \sqrt{13} + 8 \approx 19.2111 \text{ FT}$$

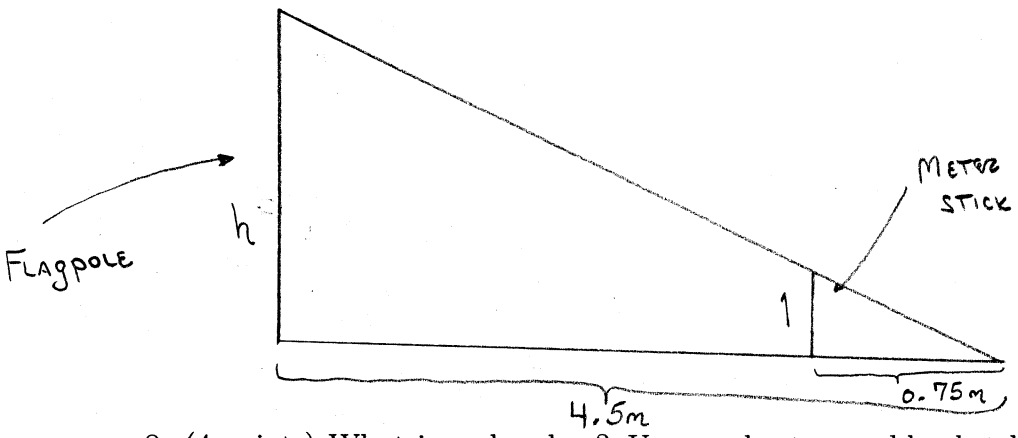
$$\frac{19.2111 \text{ FT}}{1} \times \frac{12 \text{ IN}}{1 \text{ FT}} \times \frac{2.54 \text{ cm}}{1 \text{ IN}} = 585.5544 \text{ cm}$$

$$\approx \boxed{5.86 \text{ m}}$$

6. (3 points) What is the name of a polygon with 7 sides? What is the measure of each interior angle of a regular 7-sided polygon?

Heptagon --- each interior angle measures $\frac{(7-2)180^\circ}{7} \approx \boxed{128.57^\circ}$

7. (3 points) On a sunny day a flagpole casts a 4.5-m shadow. At the same time, a meter stick cast a 0.75-m shadow. Use similar triangles to find the height of the flagpole.



$$\frac{h}{4.5} = \frac{1}{0.75}$$

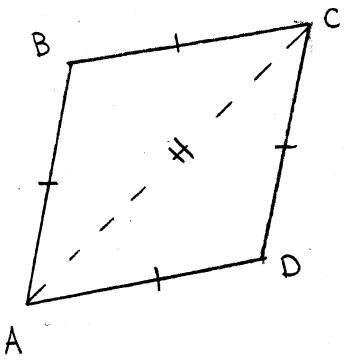
$$0.75h = 4.5$$

$$h = \frac{4.5}{0.75}$$

$$h = 6m$$

8. (4 points) What is a rhombus? Use a ruler to roughly sketch a rhombus. (It doesn't have to be perfect!) Then draw one of the diagonals. Give a convincing argument that the diagonal divides the rhombus into two congruent triangles.

A RHOMBUS IS A QUADRILATERAL WITH ALL 4 SIDES CONGRUENT.



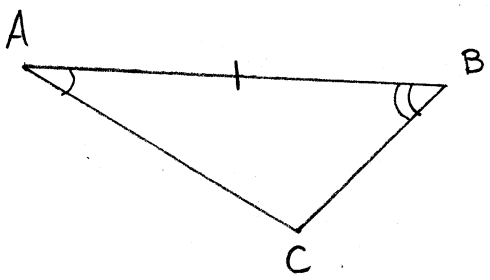
LOOK AT $\triangle ABC$ AND $\triangle CDA$

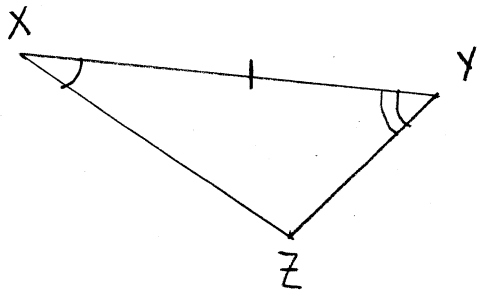
$\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong DA$
} BECAUSE THE FIGURE IS A RHOMBUS

$\overline{AC} \cong \overline{CA}$ --- SHARED SIDE

$\triangle ABC \cong \triangle CDA$ BY SSS

9. (4 points) Sketch two obtuse triangles that appear to be congruent. Label them $\triangle ABC$ and $\triangle XYZ$. Indicate what must be true if the triangles are congruent by the ASA property.





TWO ANGLES AND THE INCLUDED SIDE MUST BE CONGRUENT, RESPECTIVELY, AS MARKED.

10. (5 points) Suppose A, B, and C are events of an experiment with sample space S. Further suppose that

$$P(A) = \frac{3}{8}, \qquad P(B) = \frac{5}{8}, \qquad P(C) = \frac{1}{2}.$$

(a) Find the odds against A.

$P(A) = \frac{3}{8} \Rightarrow$ ODDS AGAINST ARE $\frac{P(\bar{A})}{P(A)} = \frac{\frac{5}{8}}{\frac{3}{8}} = \boxed{\frac{5}{3}}$

(b) Find $P(A \cup C)$ if $P(A \cap C) = 0.25$

$P(A \cup C) = P(A) + P(C) - P(A \cap C) = \frac{3}{8} + \frac{1}{2} - 0.25 = \boxed{\frac{5}{8}} = 0.625$

(c) Find $P(\overline{B})$.

$P(\bar{B}) = 1 - P(B) = 1 - \frac{5}{8} = \boxed{\frac{3}{8}}$

(d) Find the odds in favor of B.

$P(B) = \frac{5}{8} \Rightarrow$ ODDS IN FAVOR ARE $\frac{P(B)}{P(\bar{B})} = \frac{\frac{5}{8}}{\frac{3}{8}} = \boxed{\frac{5}{3}}$

(e) Is it possible that B and C are mutually exclusive? Explain.

No way, $P(B) + P(C) = \frac{5}{8} + \frac{1}{2} > 1$

11. (4 points) The prices of a certain stock throughout the day are shown below.

9am — \$3.50, 11am — \$5.00, 12noon — \$6.25, 2pm — \$6.50, 3pm — \$4.00

What type of graph would be best for displaying this data? Sketch the corresponding graph on the attached graph paper.

LINE GRAPH IS BEST. SEE GRAPH PAPER.

12. (3 points) An experiment consists of selecting a letter at random from the word SPEED. List the elements of a possible sample space, and give an event with probability 4/5.

SAMPLE SPACE $= \{S, P, E, D\}$

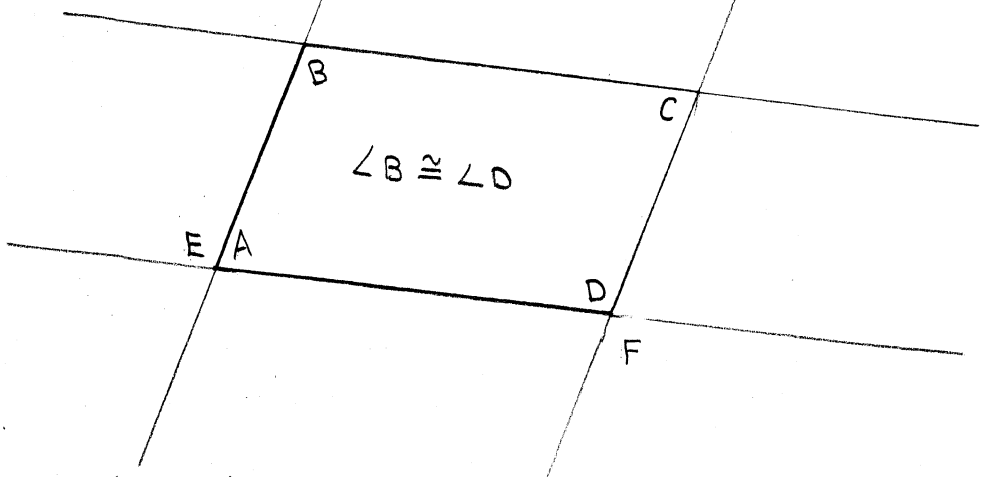
↑ THIS IS NOT A UNIFORM SAMPLE SPACE.

$A = \{P, E, D\}$

$P(A) = \frac{4}{5}$

$\angle B \cong \angle E \cong \angle F \cong \angle D$

13. (5 points) Use a ruler to roughly sketch a parallelogram. (It doesn't have to be perfect!) Extend each side to form two pairs of parallel lines. Now choose any pair of the parallelogram's opposite interior angles and prove that they are congruent.



$\angle B \cong \angle E$ BECAUSE THEY ARE ALT. INTERIOR $\angle$'s

$\angle E \cong \angle F$ BECAUSE THEY ARE ALT. EXTERIOR $\angle$'s

$\angle F \cong \angle D$ BECAUSE THEY ARE VERTICAL

14. (5 points)

(a) How many distinct points are needed to uniquely define a line?

2 POINTS

(b) How many distinct, non-collinear points are needed to uniquely define a plane?

3 POINTS

(c) What does CPCTC stand for?

CORRESPONDING PARTS OF CONGRUENT TRIANGLES ARE CONGRUENT.

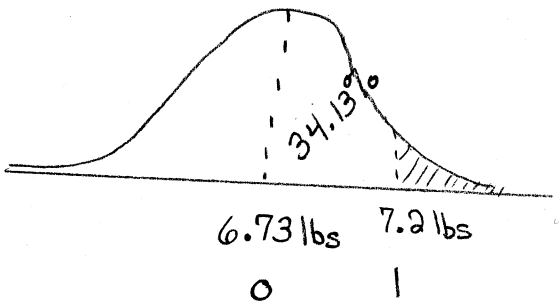
(d) What does it mean for a 2D figure to be simple?

A CURVE IS SIMPLE IF IT DOES NOT CROSS ITSELF.

(e) What are skew lines?

LINES THAT CANNOT BE IN THE SAME PLANE ARE SKEW.

15. (4 points) Biologists studying Australia's Long-Nosed Bandicoot have found that adult males have a mean weight of 6.73 lbs with a standard deviation of 0.47 lbs. Assuming bandicoot weights are normally distributed, about what percent of adult male bandicoots weigh more than 7.2 lbs?



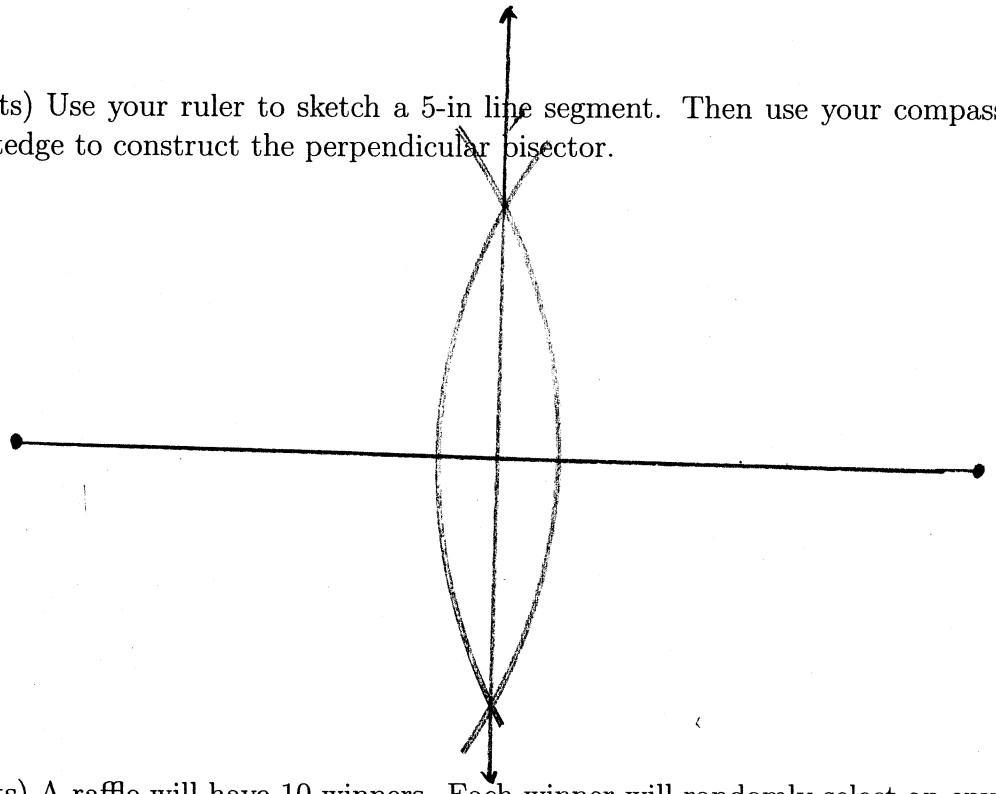
$50\% - 34.13\%$

$= \boxed{15.87\%}$

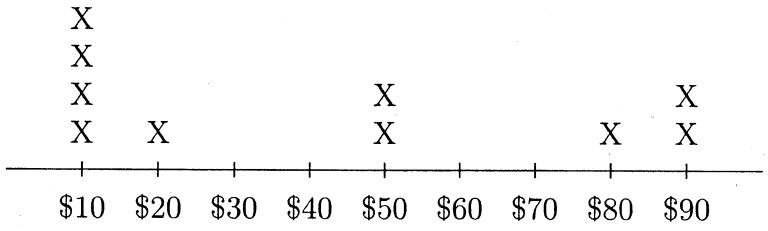
$Z = \frac{7.2 - 6.73}{0.47} = 1$

Look up 1 to get 0.3413

16. (4 points) Use your ruler to sketch a 5-in line segment. Then use your compass and straightedge to construct the perpendicular bisector.



17. (4 points) A raffle will have 10 winners. Each winner will randomly select an envelope that contains a certain amount of money. The dot plot below shows the amounts in the envelopes.

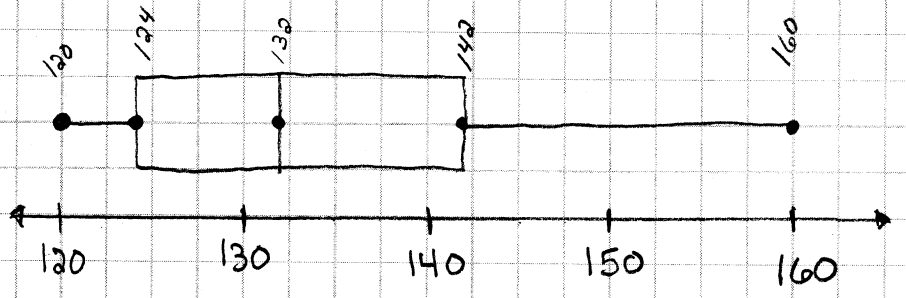


How much should the first winner expect to win? (Compute the expected value.)

$\$10\left(\frac{4}{10}\right) + \$20\left(\frac{1}{10}\right) + \$50\left(\frac{2}{10}\right)$

$+ \$80\left(\frac{1}{10}\right) + \$90\left(\frac{2}{10}\right) = \boxed{\$42}$

Boxplot for #3



Graph for #11