

# Math 206 - Test 1

February 16, 2011

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) In an experiment in which three colored marbles are drawn from a jar, the probability of drawing at least one red marble is 0.635. What is the probability of drawing no red marbles? Briefly explain your reasoning.

THE EVENTS ARE COMPLEMENTARY. PROB OF NO RED  
 $= 1 - 0.635 = 0.365$

2. (4 points) Consider the experiment of drawing one number, at random, from the box shown below.

$\sqrt{2}$	$5/3$	$-8$	$\sqrt{4}$	$7.\overline{47}$	$\pi$	$1.232332333\dots$
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- (a) What is the event of drawing an irrational number?

$$A = \{ \sqrt{2}, \pi, 1.23233\dots \}$$

- (b) What is the probability of drawing a natural number?

$$B = \{ \sqrt{4} \}$$

$$P(B) = \frac{1}{7}$$

- (c) What is the probability of drawing a rational number?

$$C = \{ \frac{5}{3}, -8, \sqrt{4}, 7.\overline{47} \}$$

$$P(C) = \frac{4}{7}$$

- (d) Is your probability in part (c) an experimental probability or a theoretical probability?

THEORETICAL

3. (3 points) Suppose  $A$  and  $B$  are mutually exclusive events with  $P(A) = 0.32$  and  $P(B) = 0.25$ . Find each of the following.

(a)  $P(A \cup B) = P(A) + P(B) = 0.32 + 0.25 = 0.57$

(b)  $P(A \cap B) = 0$  MUTUALLY EXCLUSIVE MEANS  $A \cap B = \emptyset$

(c)  $P(A \cup \bar{A}) = 1$  SINCE  $A \cup \bar{A} = \text{SAMPLE SPACE}$

4. (6 points) Solve each of the following. Use the unit rate approach for one, the scale factor approach for one, and standard cross multiplication for one. Be sure to state which approach is which.

(a) Jonathon can make 8 snowballs in the time it takes William to make 5. How many has William made by the time Jonathon has made 32?

SCALE FACTOR APPROACH: IF JONATHON MAKES 4 TIMES AS MANY SNOWBALLS, WILLIAM WILL MAKE 4 TIMES AS MANY.

$$5 \times 4 = 20$$

WILLIAM MAKES 20.

(b) If 12 donuts cost \$4.68, about how many donuts can be purchased for \$7.25?

UNIT RATE APPROACH:

$$\square \times 0.39 = 7.25$$

$$\frac{\$4.68}{12} = \$0.39 \text{ DONUTS COST } 39\text{¢ EACH.}$$

$$\frac{7.25}{0.39} \approx 18.59.$$

18 DONUTS CAN BE PURCHASED

(c) Manny can build 9 birdhouses in 2 hours. How many birdhouses can Manny build in 5 hours?

CROSS MULTIPLY:

$$\frac{9 \text{ bh}}{2 \text{ hr}} = \frac{\square}{5 \text{ hr}}$$

$$\frac{45}{2} = 22.5$$

MANNY CAN BUILD 22.5 BIRDHOUSES

5. (3 points) The bags in the front of class contain a number of colored chips. Consider the experiment of drawing a single chip from a bag. Without counting the chips in the bag, assign an experimental probability to the event of randomly selecting a red chip. Follow-up question: If I told you there were 30 chips in the bag, how many red chips do you think there are?

THERE WERE 20 CHIPS IN EACH BAG AND 7 WERE RED.

YOU SHOULD ASSIGN AN EXPERIMENTAL PROBABILITY REPEATEDLY DOING THE EXPERIMENT.

$$\text{Assign prob} = \frac{\# \text{ OF RED CHIPS SELECTED}}{\# \text{ OF TRYS.}}$$

Follow-up:  
 ↑  
 Multiply THIS BY 30 TO GET AN ESTIMATE FOR THE # OF RED CHIPS.

$$\text{Let } F = 1.2\overline{45}$$

6. (3 points) Write  $1.2\overline{45}$  as a fraction in lowest terms.

$$\begin{array}{r} 1000F = 1245.\overline{45} \\ 10F = 12.\overline{45} \\ \hline \end{array}$$

$$990F = 1233$$

$$F = \frac{1233}{990} = \boxed{\frac{137}{110}}$$

7. (3 points) Sheila and Dora worked 3.5 hours and 4.5 hours, respectively, on a programming project. All together they were paid \$176 for completing the project. How much did each earn if they were both paid the same hourly rate?

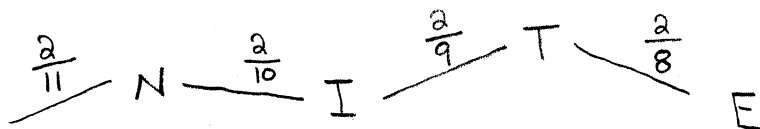
THESE ARE LOTS OF WAYS TO DO THIS PROBLEM...

SINCE BOTH ARE PAID THE SAME RATE  $r$ , WE MUST

$$\begin{array}{l} \text{HAVE} \\ \underbrace{3.5r}_{\text{SHEILA}} + \underbrace{4.5r}_{\text{DORA}} = 176 \quad \text{OR} \quad 8r = 176 \\ \Rightarrow r = 22 \end{array}$$

SHEILA MADE \$77 AND DORA

8. (3 points) Four letters are selected one at a time, without replacement, from the word INTELLIGENT. What is the probability of selecting the letters NITE in that order? MADE \$99



$$\text{Prob is } \frac{2 \times 2 \times 2 \times 2}{11 \times 10 \times 9 \times 8} = \boxed{\frac{16}{7920}}$$

$$= \frac{1}{495}$$

9. (2 points) Write the number 92.30045 in expanded form.

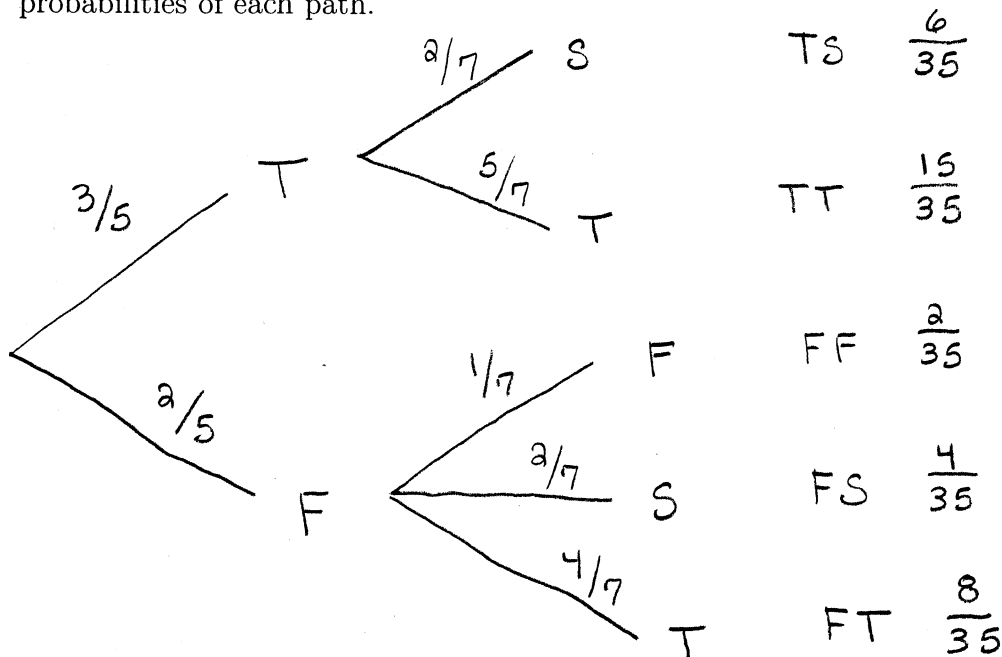
$$9 \times 10^1 + 2 \times 10^0 + 3 \times 10^{-1} + 4 \times 10^{-4} + 5 \times 10^{-5}$$

10. A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

T T T F F

S S T T T T

- (a) (3 points) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



- (b) (1 point) What is the probability that the letter T is selected from box 2?

$$P(\{TT, FT\}) = \frac{15}{35} + \frac{8}{35} = \boxed{\frac{23}{35}}$$

- (c) (1 point) What is the probability of drawing the letter S or the letter T from box 2?

$$P(\{TS, TT, FS, FT\}) = \frac{6}{35} + \frac{15}{35} + \frac{4}{35} + \frac{8}{35} = \boxed{\frac{33}{35}}$$

- (d) (1 point) How is your probability in part (c) related to the probability of drawing an F from box 2?

THE CORRESPONDING EVENTS ARE

COMPLEMENTARY:  $P(\{FF\}) = 1 - \frac{33}{35} = \frac{2}{35}$

11. (3 points) Tom can paint 4 walls in 2 hours while John can paint 2 walls in 4 hours. Working together, how long will it take them to paint 6 walls?

$$\begin{array}{l}
 \text{Tom: } \frac{4 \text{ WALLS}}{2 \text{ HRS}} = \frac{2 \text{ WALLS}}{1 \text{ HR}} \\
 \text{John: } \frac{2 \text{ WALLS}}{4 \text{ HRS}} = \frac{0.5 \text{ WALLS}}{1 \text{ HR}}
 \end{array}
 \left. \vphantom{\begin{array}{l} \text{Tom:} \\ \text{John:} \end{array}} \right\} \text{ Together: } \frac{2.5 \text{ WALLS}}{1 \text{ HR}}$$

$$\frac{6}{2.5} = \boxed{2.4 \text{ HR}}$$

$$\frac{2.5 \text{ WALLS}}{1 \text{ HR}} = \frac{6 \text{ WALLS}}{\boxed{\phantom{00}}}$$

12. (4 points) (DO THIS PROBLEM WITHOUT USING YOUR CALCULATOR OR DOING DIVISION.) Tell whether the decimal form of each number terminates or repeats. If the decimal form terminates, find the decimal form. Show your work and explain your reasoning.

(a)  $\frac{11}{125} = \frac{11}{5^3} \cdot \frac{2^3}{2^3} = \frac{88}{1000} = \boxed{0.088}$

TERMINATES

(b)  $\frac{28}{350} = \frac{4}{50} \cdot \frac{2}{2} = \frac{8}{100} = \boxed{0.08}$

TERMINATES

13. (2 points) The ratio of the length to the width of a picture frame is 7 : 5. Find the width of the frame if its length is 32 in. After you have found the width, write all four different proportions that model the problem situation.

$$\frac{7}{5} = \frac{32}{x} \Rightarrow 7x = 160$$

$$\boxed{x \approx 22.86 \text{ IN}}$$

INSTEAD OF REWRITING  
THIS, I'LL JUST USE  
"X" IN THE FOUR 5  
PROPORTIONS

$$\textcircled{1} \frac{7}{5} = \frac{32}{x}$$

$$\textcircled{2} \frac{5}{7} = \frac{x}{32}$$

$$\textcircled{3} \frac{7}{32} = \frac{5}{x}$$

$$\textcircled{4} \frac{32}{7} = \frac{x}{5}$$

14. (3 points) Consider the number  $\frac{1}{67}$ .

(a) Circle each set of numbers to which this number belongs.

Whole Numbers    Integers    Rational Numbers

Irrational Numbers    Real Numbers

(b) Does the decimal form of this number repeat, terminate, or neither? How do you know?

REPEATS.  $\frac{1}{67}$  IS IN LOWEST TERMS

AND 67 IS PRIME. THE DENOMINATOR DOES NOT HAVE FACTORS OF 2'S OR 5'S.

(c) To thirty-one decimal places,  $\frac{1}{67} = 0.0149253731343283582089552238805$ . Are you surprised that it has not terminated or begun to repeat after this many places? Explain your answer.

NOT SURPRISED, THE REPETEND COULD POSSIBLY HAVE 66 DIGITS.

15. (3 points) A letter is selected at random from the word EYJAFJALLAJOKULL.

(a) Give examples of two equally likely events.

SAMPLE SPACE =  $\{E, Y, J, A, F, L, O, K, U\}$

$\{O\}$  AND  $\{Y\}$  BOTH HAVE PROBABILITY  $\frac{1}{16}$

(b) Give examples of two mutually exclusive events.

$\{A, E, O\}$      $\{Y, J, F, L\}$

(c) Give an example of an event  $A$  such that  $P(A) > 1$ .

NOT POSSIBLE. PROBABILITIES ARE ALWAYS BETWEEN 0 AND 1.