

Math 206 - Test 1
February 19, 2014

Name key Score _____

Show all work. Supply explanations where necessary.

1. (3 points) Suppose a letter is selected at random from the word *SENSELESSNESS*.

(a) Give a possible sample space for this experiment.

$$\{S, E, N, L\}$$

(b) Is your sample space uniform? Explain.

NO, THE OUTCOMES ARE NOT EQUALLY LIKELY:

$$P(\{S\}) = \frac{6}{13}, P(\{E\}) = \frac{4}{13}, P(\{N\}) = \frac{2}{13}, P(\{L\}) = \frac{1}{13}$$

(c) Describe an event that has a probability greater than 0.25 but less than 0.5. Give the probability of your event.

THE EVENT OF SELECTING THE LETTER E.

$$\text{HAS PROBABILITY } \frac{4}{13} \approx 0.308$$

2. (3 points) Consider the number $\frac{5}{17}$.

(a) Use your calculator to find the decimal form. Round to the nearest ten-thousandth.

$$\frac{5}{17} \approx \boxed{0.2941}$$

(b) Does the decimal form repeat or terminate?

SINCE $\frac{5}{17}$ IS IN LOWEST TERMS AND 17 IS PRIME (NOT 2 OR 5),

THE DECIMAL FORM MUST REPEAT.

(c) Your calculator should show ten digits of the decimal form. Are you surprised that it has neither terminated nor begun to repeat after those ten digits? Explain.

NO, I AM NOT SURPRISED BECAUSE THE

REPEATED COULD HAVE UP TO 16 DIGITS.

3. (3 points) Four letters are selected one at a time, without replacement, from the word INTELLIGENT. What is the probability of selecting the letters LING in that order?

$$\frac{2}{11} \text{ L } \frac{2}{10} \text{ I } \frac{2}{9} \text{ N } \frac{1}{8} \text{ G}$$

$$\text{Prob is } \frac{2}{11} \times \frac{2}{10} \times \frac{2}{9} \times \frac{1}{8}$$

4. (4 points) Consider the experiment of drawing one number at random from the box shown below.

$$= \frac{8}{7920}$$

$\sqrt{16}$	$-\frac{2}{17}$	0	$\sqrt{5}$	π	$3.\overline{14}$	5.131331333...
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- (a) What is the event of drawing an irrational number?

$$\{\sqrt{5}, \pi, 5.131331333...\}$$

- (b) What is the probability of drawing a natural number?

$$P(\{\sqrt{16}\}) = \frac{1}{7}$$

- (c) What is the probability of drawing a rational number?

$$P(\{\sqrt{16}, -\frac{2}{17}, 0, 3.\overline{14}\}) = \frac{4}{7}$$

- (d) What are the odds in favor of drawing a number with a repeating decimal form?

$$P(\{-\frac{2}{17}, 3.\overline{14}\}) = \frac{2}{7}$$

$$\text{Odds are } \frac{2}{5}.$$

5. (2 point) Isaac is wearing his sombrero as he sits under an apple tree. The area of his sombrero is 3 ft^2 . The apple tree covers 96 ft^2 . What is the probability that Isaac will be hit by a randomly falling apple?

$$\frac{\text{Area of Hat}}{\text{Area of Tree}} = \frac{3}{96} = 0.03125$$

6. (3 points) A dart lands at random on the board shown below. The thrower wins the amount of money associated with the dart's location. What is the probability that the thrower wins more than \$7? Briefly explain your reasoning.

\$2	\$6	
	\$8	\$9
	\$4	

↑ THE SHADED REGIONS ARE ASSOCIATED WITH "MORE THAN \$7".

THE SHADED REGION IS $(\frac{1}{8} + \frac{1}{16})$ OF THE

TOTAL REGION. PROBABILITY IS $\frac{1}{8} + \frac{1}{16}$

$$= \frac{3}{16}$$

7. (4 points) Write $0.\overline{37}$ as a fraction in lowest terms.

$$\begin{aligned} F &= 0.\overline{37} \\ 100F &= 37.\overline{37} \end{aligned} \rightarrow 100F - F = 99F = 37$$

$$F = \frac{37}{99}$$

8. (3 points) A jar is filled with colored marbles. The probability of selecting a red marble is $\frac{37}{60}$. Is it possible that the probability of selecting a blue marble is $\frac{2}{5}$? Explain.

No. SELECTING BLUE AND SELECTING RED

ARE EXCLUSIVE. THE PROBABILITY

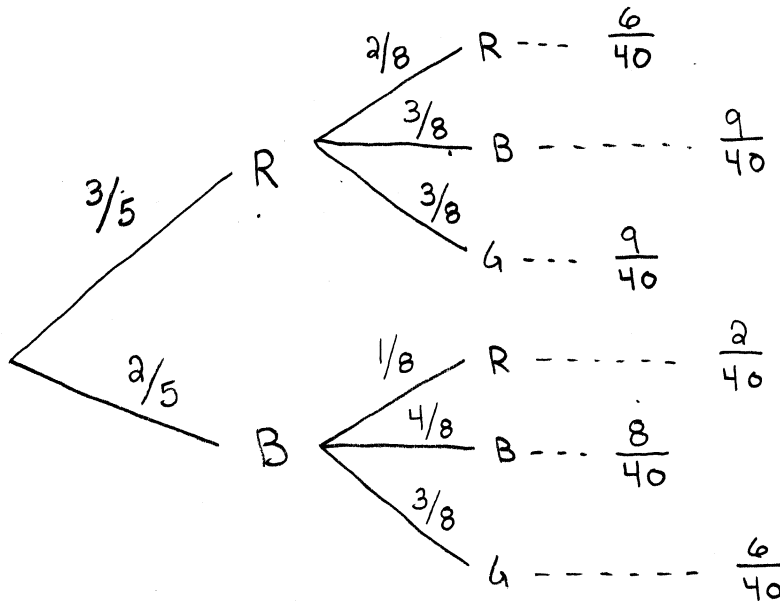
OF THE UNION IS OBTAINED BY ADDING

$$\text{THE PROBS: } \frac{37}{60} + \frac{2}{5} = \frac{61}{60} > 1.$$

A PROBABILITY CANNOT EXCEED 1.

9. Box 1 contains 3 red marbles and 2 blue marbles. Box 2 contains 1 red marble, 3 blue marbles, and 3 green marbles. A marble is selected at random from Box 1 and placed into Box 2. Then a marble is selected from Box 2.

(a) (5 points) Sketch the complete tree diagram for this experiment. Include the probabilities of each path:



(b) (1 point) What is the probability that a blue marble is selected from Box 2?

$$\frac{9}{40} + \frac{8}{40} = \boxed{\frac{17}{40}}$$

(c) (1 point) Use your probability from part (b) to determine the probability that a non-blue marble is selected from Box 2.

$$1 - \frac{17}{40} = \boxed{\frac{23}{40}}$$

(d) (2 points) What is the probability of selecting a red marble from Box 1 or a green marble from Box 2?

$$\frac{3}{5} + \frac{6}{40} = \frac{24}{40} + \frac{6}{40} = \boxed{\frac{30}{40}}$$

10. (2 points) The odds in favor of the event A are $20/7$. What is $P(\bar{A})$?

$$P(A) = \frac{20}{27} \Rightarrow P(\bar{A}) = \frac{7}{27}$$

11. (3 points) Sam was interested in answering the following question:

If a PSC student is chosen at random, what is the probability that the student is taking a math class?

To answer his question, Sam asked 75 students as they passed through the doors, and 43 were taking a math class.

- (a) Sam assigned the probability a value of $43/75$. Is this a theoretical or experimental probability?

EXPERIMENTAL

- (b) What would Sam need to do to assign the other type of probability?

USE STUDENT RECORDS $\left(\frac{\text{NUMBER OF PSC STUDENTS TAKING MATH}}{\text{TOTAL \# OF PSC STUDENTS}} \right)$
OR SURVEY ALL STUDENTS \uparrow TO COMPUTE

12. (4 points) Without using your calculator or doing division, write each fraction as a terminating decimal.

(a) $\frac{7}{8} = \frac{7}{8} \cdot \frac{125}{125} = \frac{875}{1000} = 0.875$

$$\begin{array}{r} 125 \\ \times 7 \\ \hline 875 \end{array}$$

(b) $\frac{9}{1500} = \frac{3}{500} \cdot \frac{2}{2} = \frac{6}{1000} = 0.006$

13. (4 points) Suppose A , B , and C are events such that $P(A) = 0.26$, $P(B) = 0.55$, and $P(C) = 0$. Compute each of the following.

(a) $P(\overline{C})$

$$= 1 - P(C) = 1 - 0 = \boxed{1}$$

(b) $P(A \cup B)$ if A and B are mutually exclusive

$$P(A \cup B) = P(A) + P(B) = 0.26 + 0.55 = \boxed{0.81}$$

(c) $P(A \cap B)$ if $P(A \cup B) = 0.67$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.81 - 0.67 = \boxed{0.14}$$

(d) The odds against B

$$P(\overline{B}) = 1 - 0.55 = 0.45$$

ODDS AGAINST ARE

$$\frac{45}{55}$$

OR $\frac{9}{11}$

14. (3 points) Each situation below describes a multistage experiment. Determine the best number of stages for each.

(a) A pair of dice are rolled.

TWO STAGES --- ONE FOR EACH DIE

(b) A marble is selected from one jar and placed into a second jar. Then two marbles are selected from the second jar.

THREE STAGES -- ONE FOR 1ST MARBLE, ONE FOR EACH OF NEXT TWO MARBLES

(c) A small bag contains a variety of coins. Four coins are selected at random.

FOUR STAGES --- ONE FOR EACH COIN.