

**Math 206 - Test 1**  
February 11, 2015

Name key Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations where necessary.

1. (3 points) Without using your calculator or using long division, write  $\frac{13}{125}$  in decimal form.

$$\frac{13}{125} = \frac{13}{5^3} \cdot \frac{2^3}{2^3} = \frac{13 \cdot 8}{10^3} = \frac{104}{1000} = \boxed{0.104}$$

2. (3 points) Without attempting to actually compute the decimal value of  $\frac{49}{350}$ , determine whether the fraction has a terminating or repeating decimal form. Explain your reasoning.

$$\frac{49}{350} = \frac{7}{50} = \frac{7}{2 \cdot 5^2}$$

REDUCE TO  
LOWEST  
TERMS

SINCE PRIME FACTORIZATION HAS  
FACTORS OF ONLY 2'S & 5'S, THE DECIMAL  
FORM TERMINATES.

3. (2 points) Without attempting to actually compute the decimal value of  $\frac{9}{19}$ , explain why its decimal form cannot be 0.4736842105263157894.

$\frac{9}{19}$  REPEATS WITH A REPEATED OF 18 OR FEWER DIGITS.

THE GIVEN REPEATED HAS 19 DIGITS, WHICH  
CANNOT BE.

4. (2 points) Give the decimal form of an irrational number between 8.34 and 8.345.

8.3410110111011110...

NEITHER REPEATS NOR TERMINATES.

5. (2 points) Is  $\sqrt{145}$  rational or irrational? Explain how you know.

$\sqrt{145}$  IS IRRATIONAL, BECAUSE 145 IS NOT A PERFECT SQUARE.

6. (3 points) Write  $0.\bar{5}$  as a fraction in lowest terms.

$$\begin{aligned} F &= 0.\bar{5} \\ 10F &= 5.\bar{5} \end{aligned} \quad \left. \vphantom{\begin{aligned} F &= 0.\bar{5} \\ 10F &= 5.\bar{5} \end{aligned}} \right\} \begin{aligned} 10F - F &= 5.\bar{5} - 0.\bar{5} \\ 9F &= 5 \end{aligned}$$
$$F = \frac{5}{9}$$

7. (3 points) The odds **against** Jasper's Gem winning the horse race are 5 to 2. What is the probability that the horse wins?

$$\begin{aligned} \text{ODDS AGAINST ARE } \frac{5}{2} &\Rightarrow \text{ODDS IN FAVOR ARE } \frac{2}{5} \\ &\Rightarrow \text{PROB IS } \frac{2}{2+5} = \frac{2}{7} \end{aligned}$$

8. (3 points) A single letter is selected at random from the word *bumfuzzle*. Let  $E$  be the event of selecting the letter  $e$ , and let  $V$  be the event of selecting a vowel. Compute each of the following.

(a)  $P(E|V)$

$$= \text{PROB OF } \{e\} \text{ GIVEN A VOWEL} = \frac{1}{3}$$

(b)  $P(E|\bar{V})$

$$= \text{PROB OF } \{e\} \text{ GIVEN A CONSONANT} = \frac{0}{6} = 0$$

(c)  $P(V|E)$

$$= \text{PROB OF } \{u, e\} \text{ GIVEN } \{e\} = \frac{1}{1} = 1$$

9. A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box and placed into the third box. Then a letter is selected at random from the third box.

○ ○ ○ ○ X

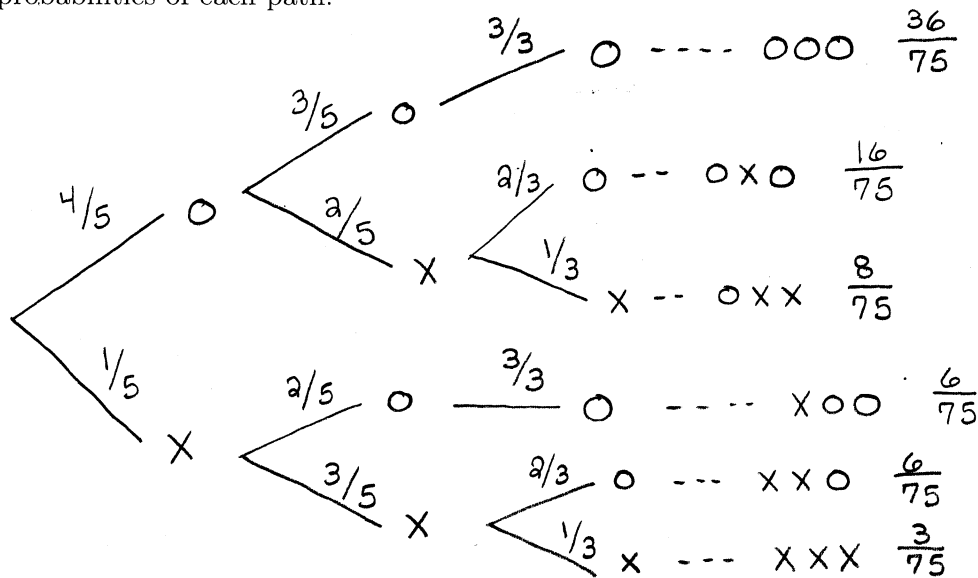
○ ○ X X

○ ○

- (a) (1 point) How many stages does this experiment have?

3 STAGES -- 1 FOR EACH SELECTION OF A LETTER

- (b) (5 points) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



- (c) (1 point) What is the probability that the letter O is selected from box 3?

$$\frac{36}{75} + \frac{16}{75} + \frac{6}{75} + \frac{6}{75} = \boxed{\frac{64}{75}}$$

- (d) (1 point) What is the sum of all the probabilities of the paths?

They must sum to  $\boxed{1}$

- (e) (1 point) Explain how your last two answers can be used to determine the probability that an X is selected from box 3.

THIS EVENT IS THE COMPLEMENT OF THE EVENT IN (C).

ITS PROB MUST BE

$$1 - \frac{64}{75} = \boxed{\frac{11}{75}}$$

10. (3 points) When a thumbtack is dropped, it will land point up or point down. An experiment was repeated 80 times with the following results: point up 56 times, point down 24 times.

(a) What value would you assign to the probability of the tack landing point down?

$$\frac{\# \text{ POINT DOWN}}{\text{TOTAL \#}} = \boxed{\frac{24}{80}}$$

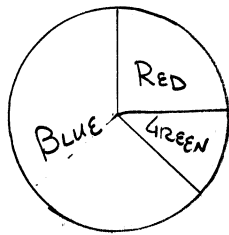
(b) Is your probability theoretical or experimental?

EXPERIMENTAL

(c) What would you do if you wanted a reasonably good estimate for the other type of probability?

DO THE EXPERIMENT A VERY LARGE NUMBER OF TIMES. THE EXPERIMENTAL PROB WILL GET CLOSE TO THE THEO PROB.

11. (3 points) Design a spinner that has only three different sections, colored red, blue, and green, so that the probability of red is 25%, the probability of blue is 60%, and the probability of green is 15%. Explain your reasoning.



$\frac{1}{4}$  OF SPINNER IS RED

$\frac{2}{3}$  IS BLUE

REST IS GREEN

PROBABILITY = PROPORTION COLORED.

12. (3 points) Consider the two different experiments: (1) select a letter from *racecar*, (2) select a letter from *sample*. Which experiment has a uniform sample space? Explain.

{ r, a, c, e }

LETTERS ARE

NOT EQUALLY LIKELY

NOT UNIFORM.

{ s, a, m, p, l, e }

LETTERS ARE EQUALLY

LIKELY.

UNIFORM

13. (3 points) A jar is filled with coins. The probability of selecting a penny is  $\frac{7}{8}$ . Is it possible that the probability of selecting a quarter is  $\frac{1}{5}$ ? Explain.

No, SELECTING A PENNY & SELECTING A QUARTER

ARE EXCLUSIVE EVENTS WHOSE UNION WOULD HAVE A PROBABILITY OF  $\frac{7}{8} + \frac{1}{5} = 1.075$ . ← PROBS CANNOT EXCEED 1.

14. (5 points) Suppose  $A$  and  $B$  are events such that  $P(A) = 0.85$ ,  $P(B) = 0.55$ , and  $P(A \cup B) = 0.93$ .

- (a) Determine  $P(A \cap B)$ .

$$P(A \cap B) = 0.85 + 0.55 - 0.93 = \boxed{0.47}$$

- (b) Determine  $P(\bar{B})$ .

$$1 - 0.55 = \boxed{0.45}$$

- (c) Find the odds in favor of  $A$ .

$$\frac{0.85}{0.15} = \frac{85}{15} = \boxed{\frac{17}{3}}$$

- (d) Determine  $P(B|A)$ .

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.47}{0.85} \approx \boxed{0.55294}$$

- (e) Are  $A$  and  $B$  independent? Explain.

No, BECAUSE  $P(B|A) \neq P(B)$ ,

BUT THESE PROBS ARE

PRETTY CLOSE.

15. (3 points) Give an example of each of the following or say that it is impossible

(a) A whole number that is not a natural number

0 IS THE ONLY SUCH NUMBER.

(b) An integer that is not a rational number

IMPOSSIBLE. EVERY INTEGER IS RATIONAL.

(c) A number that is real and irrational

$\sqrt{2}$ ,  $\pi$ ,  $0.101100111000\dots$

16. (2 pts extra credit) Determine the repeating decimal form of the product  $0.75 \times 0.\overline{5}$  by (1) writing each number as a fraction, (2) multiplying the fractions, and then (3) using division to write your result in decimal form.

$$0.75 = \frac{3}{4}$$

$$0.\overline{5} = \frac{5}{9} \quad (\text{SEE PROBLEM 6})$$

$$\frac{3}{4} \times \frac{5}{9} = \frac{5}{12}$$

$$\begin{array}{r} 0.4166\dots \\ 12 \overline{) 5.00000} \\ \underline{48} \phantom{00} \\ 20 \phantom{00} \\ \underline{12} \phantom{00} \\ 80 \phantom{00} \\ \underline{72} \phantom{00} \\ 80 \phantom{00} \\ \underline{72} \phantom{00} \\ 8 \phantom{00} \\ \vdots \end{array}$$

$$0.75 \times 0.\overline{5} = 0.41\overline{6}$$