

**Math 206 - Final Exam**  
May 13, 2015

Name key  
Score \_\_\_\_\_

Show all work. Supply explanations where necessary. Use only a compass and a straightedge for constructions. For each construction, the steps you follow must be apparent.

1. (5 points) An experiment consists of tossing a penny, a nickel, and a dime simultaneously.

(a) List the elements of an equally likely sample space. (There are 8 elements in the sample space.)

IN PENNY, NICKEL, DIME ORDER...

$$S = \{ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT \}$$

(b) List the event (not the probability) of obtaining a tail on the nickel.

$\{ HTH, HTT, TTH, TTT \}$  T IN 2<sup>ND</sup> SPOT

(c) What is the probability of obtaining a tail on the nickel?

$$\frac{4}{8}$$

(d) Is your answer for part (c) an experimental probability or a theoretical probability?

THEORETICAL

2. (4 points) The mean score for 25 of 27 tests is 80. The other two scores are 30 and 35.

(a) What is the mean of all twenty-seven test scores?

$$\frac{25(80) + 30 + 35}{27} = \frac{2065}{27} \approx 76.5$$

(b) Is it possible to find the median test score with the given information? Explain.

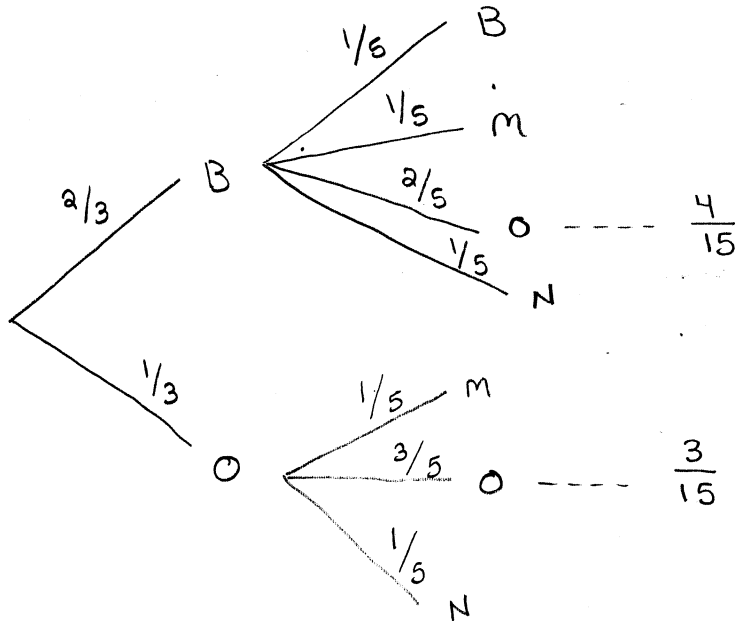
NO, THE MEDIAN IS THE 14<sup>TH</sup> SCORE  
WHEN THE SCORES ARE ARRANGED IN ORDER.  
WE HAVE NO INFO ABOUT THAT SCORE.

3. (4 points) A letter is selected at random from the first box and placed into the second box. A letter is then selected from the second box.

B O B

M O O N

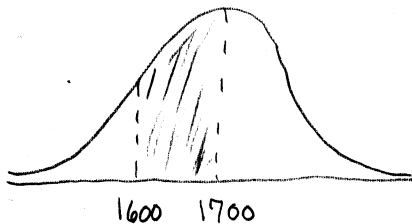
In this two-stage experiment, what are the odds in favor of selecting the letter "O" from the second box?



PROB OF "O" FROM  
2ND IS  $\frac{7}{15}$

ODDS ARE  
 $\frac{7}{8}$

4. (4 points) A certain brand of light bulb has a mean lifetime of 1700 hr with a standard deviation of 150 hr. Assuming the lifetimes are normally distributed, about how many in an order of 5000 will have a lifetime of between 1600 hr and 1700 hr?



$$5000 \times \text{normalcdf}(1600, 1700, 1700, 150)$$

$$5000 \times 0.247507\dots$$

$$\approx 1238$$

5. (4 points) A game consists of rolling a regular die with prizes awarded as follows:

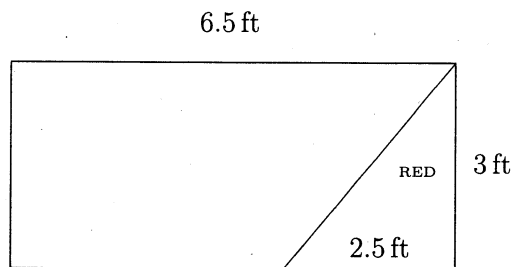
- Roll a 1 and win \$5
- Roll a 2, 4, or 6 and win \$2
- Roll a 3 or 5 and win \$1

If the game costs \$3 to play, how much money should one expect to gain or lose on average with each play?

$$E = 5\left(\frac{1}{6}\right) + 2\left(\frac{3}{6}\right) + 1\left(\frac{2}{6}\right) = \frac{13}{6} = \$2\frac{1}{6}$$

You SHOULD EXPECT TO LOSE  $\$ \frac{5}{6}$  PER PLAY

6. (5 points) A rectangular piece of plywood that measures 6.5 ft by 3 ft has a triangular region painted red on one side.



- (a) Some children laid the piece of plywood in their driveway and pitched pennies at it. They found that for every four pennies they pitched, one landed in the red triangle. If we assign a value of  $1/4$  to the probability of a penny landing in the red triangle, is this probability a theoretical or an experimental probability? Explain.

THIS PROB IS EXPERIMENTAL --- IT WAS ASSIGNED BY COUNTING OBSERVATIONS.

- (b) By computing areas, assign a value to the probability of a penny landing in the red triangle. Is this probability a theoretical or an experimental probability?

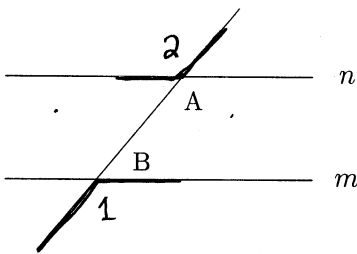
$$\text{Area of Rect} = 3 \times 6.5 \text{ ft}^2 = 19.5 \text{ ft}^2$$

$$\text{Area of Triangle} = \frac{1}{2} \times 2.5 \times 3 \text{ ft}^2 = 3.75 \text{ ft}^2$$

$$\text{Prob of Red} = \frac{3.75}{19.5} \approx 19\%$$

THIS PROB IS THEORETICAL!

7. (4 points) In the following figure  $n \parallel m$ .



(a) In the figure, label a pair of alternate exterior angles.

$\angle 1$  AND  $\angle 2$  ARE ALT EXT  $\angle$ 'S

(b) Find  $x$  if  $m(\angle A) = 2x + 4$  and  $m(\angle B) = 3x - 14$ .

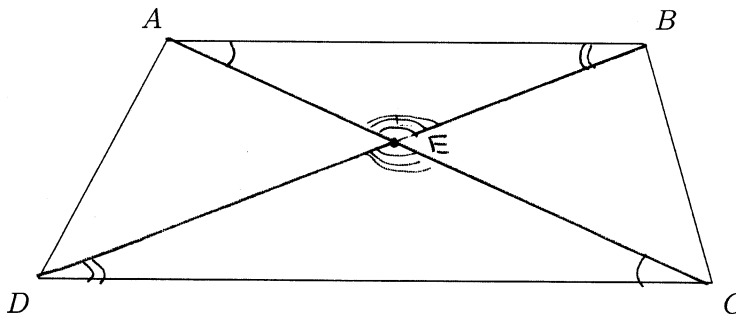
$$m(\angle A) + m(\angle B) = 180^\circ$$

$$2x + 4 + 3x - 14 = 180^\circ$$

$$5x - 10 = 180^\circ \Rightarrow 5x = 190^\circ \Rightarrow$$

$$x = 38^\circ$$

8. (4 points) In the following quadrilateral,  $\overline{AB} \parallel \overline{CD}$ .



(a) What is the specific name for the quadrilateral shown above?

TRAPEZOID

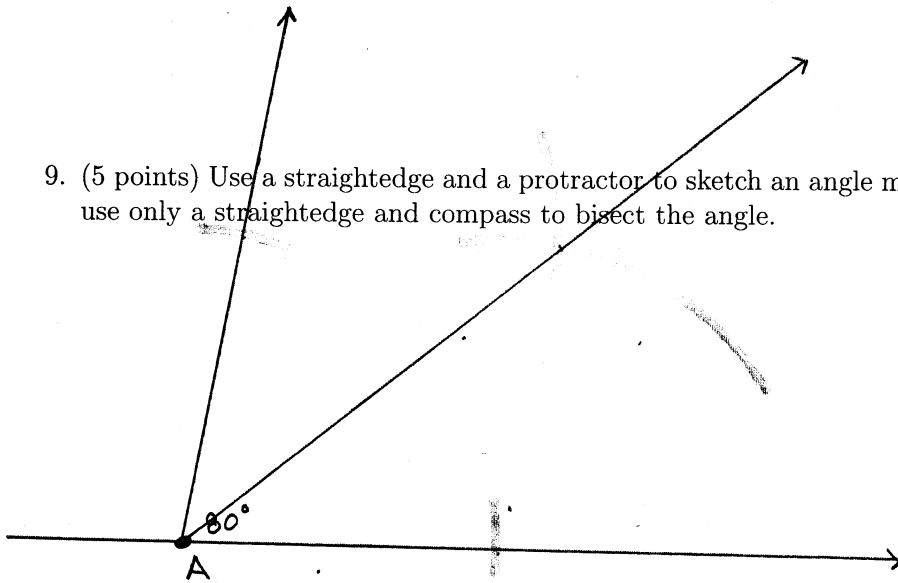
(b) Sketch the diagonals of the quadrilateral. Once you have sketched the diagonals, find two similar triangles within the figure. How do you know they are similar?

$\triangle AEB \sim \triangle CED$  By AAA prop.

$\left. \begin{array}{l} \angle BAE \cong \angle DCE \\ \angle ABE \cong \angle CDE \end{array} \right\}$  CONGRUENT ALT INT ANGLES  
 4

$\angle AEB \cong \angle CED$  (VERTICAL  $\angle$ 'S)

9. (5 points) Use a straightedge and a protractor to sketch an angle measuring  $80^\circ$ . Then use only a straightedge and compass to bisect the angle.



10. (4 points) An interior angle of a convex hexagon measures  $100^\circ$ . All of the other interior angles have equal measures (but not  $100^\circ$ ). Find the measures of the interior and exterior angles.

INT ANGLES ADD UP TO  $4(180^\circ) = 720^\circ$

$$5x = 720^\circ - 100^\circ = 620^\circ$$

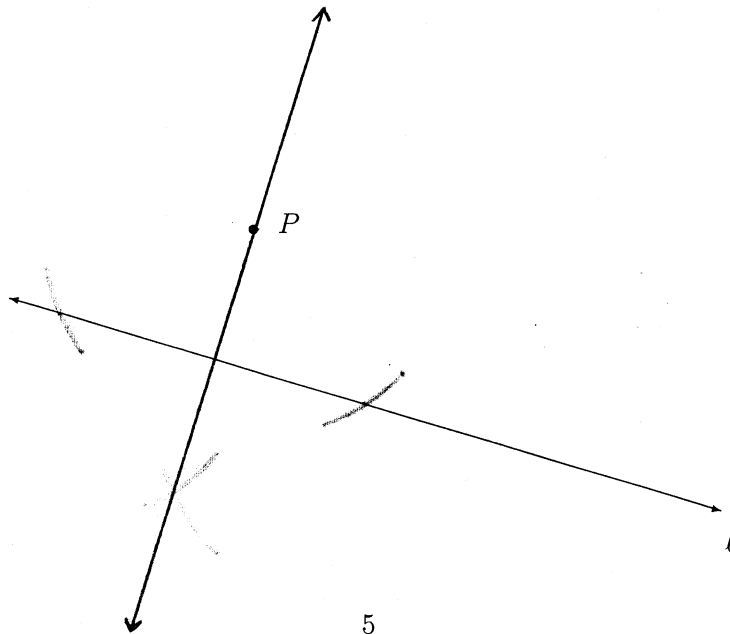
$$x = 124^\circ$$

INTERIOR  $\angle$ 's MEASURE  $100^\circ$  &  $124^\circ$

THIS MAKES THE EXT  $\angle$ 's  
MEASURE  $80^\circ$  &  $56^\circ$

(NOTICE THAT  $5(56^\circ) + 80^\circ = 360^\circ$  AS EXPECTED.)

11. (4 points) Use only a compass and straightedge to construct a line through  $P$  perpendicular to  $l$ .



12. (6 points) Suppose  $A$ ,  $B$ , and  $C$  are events such that  $P(A) = 0.57$ ,  $P(B) = 0.24$ , and  $P(C) = 0.99$ .

(a) Find  $P(\bar{C})$ .

$$P(\bar{C}) = 1 - P(C) = 1 - 0.99 = \boxed{0.01}$$

(b) Find  $P(A \cup B)$  if  $A$  and  $B$  are mutually exclusive.

$$P(A \cup B) = P(A) + P(B) = \boxed{0.81}$$

(c) Find  $P(A \cap B)$  if  $P(A \cup B) = 0.5$ .

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.5 = 0.81 - P(A \cap B) \Rightarrow \boxed{P(A \cap B) = 0.31}$$

(d) Is it possible that  $C$  and  $B$  are mutually exclusive? Explain.

$$\text{No BECAUSE IF SO } P(C \cup B) = 0.24 + 0.99 > 1$$

PROBS CANNOT EXCEED 1

13. (6 points) Fill in the blank with the correct word.

(a) Lines that share a common point are called CONCURRENT lines.

(b) A curve made up entirely of straight line segments is called a POLYGONAL curve.

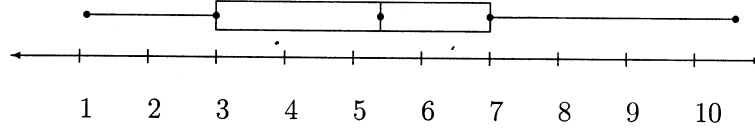
(c) Points that lie on the same line are said to be COLLINEAR.

(d) A Simple curve is a curve that does not cross itself.

(e) The angle between two planes is called a DIHEDRAL angle.

(f) Two coplanar lines that have exactly one point in common are called INTERSECTING lines.

14. (6 points) The boxplot shown below describes a certain collection of data. Find approximate values for the median, lower and upper quartiles, and the interquartile range. Based on your approximations, what would be the cutoff values for outliers?



MEDIAN  $\approx 5.3$

$Q_1 \approx 3$

$Q_3 \approx 7$

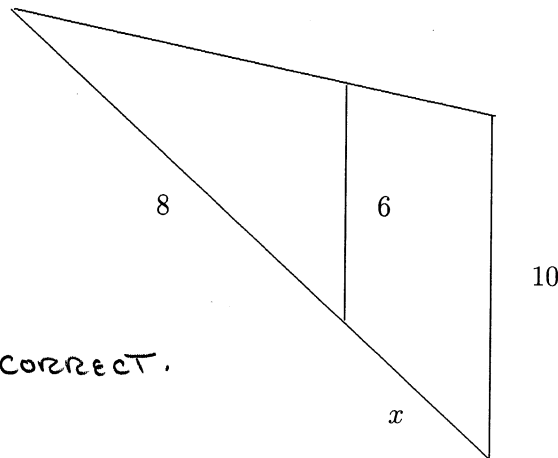
$IQR \approx 7 - 3 = 4$

CUTOFFS...

$Q_1 - 1.5 \times IQR \approx -3$

$Q_3 + 1.5 \times IQR \approx 13$

15. (4 points) Trudy was trying to find the value of  $x$  in the following diagram. She knew that there were similar triangles shown in the diagram, so she set up and solved the following proportion:  $\frac{6}{10} = \frac{8}{x}$ . Her answer didn't seem quite right, but she knew that these diagrams are not always drawn to scale. Is Trudy correct? Carefully explain your reasoning.



Trudy is NOT correct.

SHE SHOULD HAVE THE

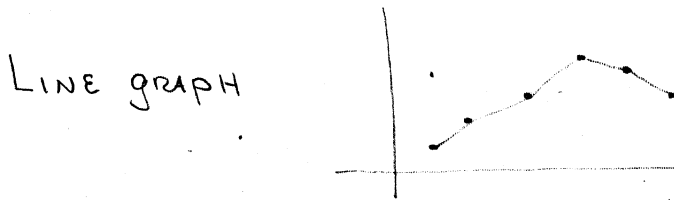
PROPORTION

$$\frac{6}{10} = \frac{8}{8+x}$$

THIS gives  $x = \frac{32}{6} = 5.\bar{3}$

16. (6 points) For each of the following situations, tell which type of graph would best display the data. Choose from *line plot*, *bar graph*, *histogram*, *line graph*, *stem-and-leaf plot*, or *circle graph*. Give a brief explanation to support your answer.

- (a) Sarah has been keeping track of gas prices. She would like to make a graph showing how the prices have changed over the last two months.



LINE GRAPHS ARE GOOD FOR SHOWING NUMERICAL TRENDS IN TIME.

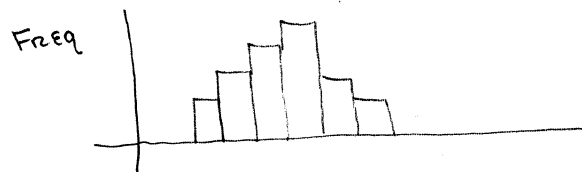
- (b) A candidate's campaign fund has come from donations in six different categories. The candidate would like to make a graph showing how the entire fund is divided up among the six categories.

CIRCLE GRAPH / PIE CHART

THESE ARE GOOD AT SHOWING PORTIONS OF A WHOLE.

- (c) The National Center for Health Statistics keeps detailed records on the births and deaths of US residents. What type of graph would best display the ages at death of US residents in 2012?

HISTOGRAM -- AGES ARE CONTINUOUS DATA.



IF WE WISH TO KNOW HOW MANY DEATHS AT CERTAIN AGES, A HISTOGRAM IS BEST.