## Solving First-Order, Linear ODEs

It is common in mathematics to multiply both sides of an equation by an expression that will in some way simplify the equation. In solving a first-order, linear ODE,

$$
y^{\prime}(x)+p(x) y(x)=q(x)
$$

we will use the same idea. We seek an integrating factor, $\mu(x)$, such that

$$
(\mu y)^{\prime}=\mu q .
$$

Let's see what $\mu(x)$ should look like...

We start with

$$
y^{\prime}+p y=q .
$$

Multiply both sides by $\mu(x)$ :

$$
\mu y^{\prime}+\mu p y=\mu q .
$$

We want the left-hand side to be equal to $(\mu y)^{\prime}$ :

$$
(\mu y)^{\prime}=\mu y^{\prime}+\mu^{\prime} y=\mu y^{\prime}+\mu p y
$$

It follows that

$$
\mu^{\prime} y=\mu p y \quad \text { or } \quad \mu^{\prime}=\mu p
$$

And it is easy to see that the last equation is satisfied by

$$
\mu(x)=e^{\int p(x) d x} .
$$

## Solving 1st order, linear ODEs

- Given $y^{\prime}(x)+p(x) y(x)=q(x)$
- Find the integrating factor $\mu(x)=e^{\int p(x) d x}$
- The solution follows from

$$
\mu(x) y(x)=\int \mu(x) q(x) d x
$$

(Don't forget your constant of integration!)

- Or, if you are given an initial condition...

$$
\mu(x) y(x)=\mu\left(x_{0}\right) y\left(x_{0}\right)+\int_{x_{0}}^{x} \mu(t) q(t) d t
$$

