Math 216 - Quiz 1

Name _____

September 1, 2010

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) In radioactive decay, the number of nuclei, N, decays according to the differential equation

$$\frac{dN}{dt} = kN,$$

where k is the decay constant and t is time. We have already solved this DE to get the exponential growth/decay model $N(t) = Ce^{kt}$. Use this model to solve the following problem.

The half-life of carbon-14 is approximately 5700 years. One of the Dead Sea scrolls found in 1947 contained 76% of its initial amount of carbon-14. About how old was the scroll?

2. (3 points) Consider the differential equation $y' = 1 - y^2$.

(a) Use the attached Sage code or another online tool to plot the direction field for the DE. Without solving the DE, draw a rough sketch of the solution curve through $(0, 0)$. What are the limits of your solution as $x \to \pm\infty$?

(b) Solve the differential equation along with the initial condition $y(0) = 0$.

3. (3 points) The slope m of a curve is 0 where the curve crosses the y-axis, and in general,

$$\frac{dm}{dx} = \sqrt{1+m^2}.$$

Find m as a function of x. (In order to integrate, you'll need to use a trigonometric substitution. Show all work.)

4. (2 points) Solve: $\dfrac{dy}{dx} - \dfrac{y}{x} = x^2, \quad y(1) = 3$