

# Math 216 - Quiz 4

September 29, 2010

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (2.5 points) Solve the following initial value problem.

$$(y^2 + 3xy) dx = (4x^2 + xy) dy, \quad y(1) = 1$$

$$\frac{dy}{dx} = \frac{y^2 + 3xy}{4x^2 + xy} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$\frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 + 3\left(\frac{y}{x}\right)}{4 + \left(\frac{y}{x}\right)}$$

$$u = \frac{y}{x}, \quad u + x \frac{du}{dx} = \frac{dy}{dx}$$

$$u + x \frac{du}{dx} = \frac{u^2 + 3u}{4+u}, \quad u(1)=1$$

$$x \frac{du}{dx} = \frac{u^2 + 3u}{4+u} - \frac{u(4+u)}{4+u}$$

$$x \frac{du}{dx} = \frac{-u}{4+u}$$

$$\frac{4+u}{u} du = -\frac{1}{x} dx$$

$$4 \ln|u| + u = -\ln|x| + C$$

$$u(1)=1 \Rightarrow 1 = C$$

$$4 \ln|\frac{y}{x}| + \frac{y}{x} + \ln|x| = 1$$

2. (2.5 points) Solve the following 2nd order equation.

$$y'' + 2y(y')^3 = 0$$

$$u = y', \quad y'' = u \frac{du}{dy}$$

$$u \frac{du}{dy} + 2yu^3 = 0$$

$$\frac{du}{dy} = -2yu^2, \quad u \neq 0$$

$$\frac{1}{u^2} du = -2y dy$$

$$-\frac{1}{u} = -y^2 + C$$

$$u = \frac{1}{y^2 - C}$$

$$u = 0 \Rightarrow y' = 0$$

$y' = 0$  is also a  
SOLUTION

$$y' = 0 \Rightarrow y = \text{CONST}$$

$$\frac{dy}{dx} = \frac{1}{y^2 - C}$$

$$(y^2 - C) dy = dx$$

$$\boxed{\frac{1}{3}y^3 - Cy + D = X}$$

OR

3. (2.5 points) Solve the following ODE.

$$\underbrace{(2y^2 + 2y + 4x^2)}_M dx + \underbrace{(2xy + x)}_N dy = 0$$

$$\frac{\partial M}{\partial y} = 4y + 2$$

$$\frac{\partial N}{\partial x} = 2y + 1$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{4y + 2 - 2y - 1}{x(2y + 1)} = \frac{1}{x}$$

INTEGRATING FACTOR IS

$$e^{\int \frac{1}{x} dx} = e^{\ln|x|} = x, \quad x > 0$$

$$(2xy^2 + 2xy + 4x^3) dx + (2x^2y + x^2) dy = 0$$

$$\frac{\partial F}{\partial x} = 2xy^2 + 2xy + 4x^3 \Rightarrow F(x,y) = x^2y^2 + x^2y + x^4 + g(y)$$

$$\frac{\partial F}{\partial y} = 2x^2y + x^2 \Rightarrow F(x,y) = x^2y^2 + x^2y + h(x)$$

$$x^2y^2 + x^2y + x^4 = C$$

4. (2.5 points) Solve the following ODE.

$$\underbrace{(2xy)}_M dx + \underbrace{(y^2 - 3x^2)}_N dy = 0$$

$$\frac{\partial M}{\partial y} = 2x$$

$$\frac{\partial N}{\partial x} = -6x$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{8x}{-2xy} = -\frac{4}{y}$$

INTEGRATING FACTOR IS

$$e^{\int -\frac{4}{y} dy} = e^{-4 \ln|y|} = \frac{1}{y^4}$$

$$\frac{2x}{y^3} dx + \left( \frac{1}{y^2} - \frac{3x^2}{y^4} \right) dy = 0$$

$$\frac{\partial F}{\partial x} = \frac{2x}{y^3} \Rightarrow F(x,y) = \frac{x^2}{y^3} + g(y)$$

$$\frac{\partial F}{\partial y} = \frac{1}{y^2} - \frac{3x^2}{y^4} \Rightarrow F(x,y) = -\frac{1}{y} + \frac{x^2}{y^3} + h(x)$$

$$\frac{x^2}{y^3} - \frac{1}{y} = C$$